Microstepping with Nonlinear Torque Modulation for Permanent Magnet Stepper Motors

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Abstract—A new approach to position control based on microstepping with nonlinear torque modulation for permanent magnet stepper motors is studied. The proposed method comprises nonlinear torque modulation, a commutation scheme, and a nonlinear current tracking controller. To improve the position tracking performance of conventional microstepping, nonlinear torque modulation is incorporated. Commutation schemes implementing field-oriented control (FOC) are developed to construct the desired current profile for torque generation without the use of direct-quadrature (DQ) transformation. The nonlinear controller is developed to guarantee the desired currents derived by the commutation scheme. It is shown that the proposed commutation scheme of FOC is equivalent to microstepping, which has desired currents with time-varying amplitudes and electrical phase advance. For position tracking, the relationships between the voltage input and current output of each phase for the position tracking are derived. The performances of the proposed methods are validated through simulations and experiments and compared with that of conventional microstepping.

Index Terms—Field-oriented control, field weakening control, microstepping, permanent magnet stepper motor.

I. INTRODUCTION

Microstepping is used to improve the resolution and increase the motion stability of permanent magnet stepper motors (PMSMs). Microstepping for two phase stepper motor is the control method that two sinusoidal inputs with 90 electrical degrees shifted are given to the PM stepper motor. These inputs subdivide each full step electronically into a large number of smaller steps. Conventional microstepping is a control method for two phase PMSMs in which the desired currents in the form of sinusoidals shifted 90°are given to a PMSM for position tracking. The desired position is used to make the desired currents. Several feedback control methods have been studied in order to improve the current tracking performance of microstepping.

Positionings are typical applications of the stepper motor (SM) electrical drives systems. Permanent magnet (PM) stepper motor have been used in positioning applications due to their durability, high efficiency and power density as well as high torque to inertia ratio and absence of rotor winding. These inputs subdivide each full step electronically into a large number of smaller steps. Microstepping with current PI and feedforward controller has been proposed to improve the tracking performance of the desired currents. To improve microstepping in industrial applications, position feedback has also been achieved by resolvers or encoders built into PMSMs. Microstepping using a disturbance observer and a variable structure controller was proposed to guarantee the desired currents using only currents feedback. However, conventional microstepping has been limited by position tracking errors that occur during the nonzero velocity period.

To overcome this limitation, we propose a new PMSM position control approach that incorporates both microstepping and nonlinear torque modulation. The proposed method consists of nonlinear torque modulation, a commutation scheme, and a nonlinear current tracking controller. We first study why the torque modulation is required to improve the position tracking performance of conventional microstepping. It turns out that the large position tracking error appears during nonzero velocity period in the conventional microstepping since the electrical dynamics is much faster than the mechanical dynamics. Thus, nonlinear torque modulation is proposed to improve the position tracking performance of conventional microstepping via backstepping. Then, commutation schemes implementing field-oriented control (FOC) and field weakening control (FWC) are developed to construct the desired current profiles for torque generation without direct-quadrature (DQ) transformation. FOC is a control method that maintains zero direct current for torque maximization and high energy efficiency.
FWC maintains negative direct current to avoid the voltage saturation operations. The nonlinear current tracking controller is implemented to guarantee that the commutation scheme derives the desired current. Standard PM stepper motors have a relatively large step size, usually 1/200 of a revolution or 1.8°.

II. MATHEMATICAL MODEL OF PMSM AND MICROSTEPPING PERFORMANCE ANALYSIS

A. Mathematical Model

A PMSM consists of a slotted stator with two phases and a permanent magnet rotor, which has north and south poles.

Assumption 1: The detent torque is ignored since it does not significantly affect the torque produced by the PMSM. The magnetic couplings between the phases and the variation in inductance due to magnetic saturation are also ignored.

With Assumption 1, the dynamics of a PMSM can be represented in the state-space domain such that

\[
\begin{align*}
\dot{\omega} &= \omega \\
\dot{\theta} &= \frac{1}{J} \left[ - K_m i_a \sin (N_r \theta) + K_m i_b \cos (N_r \theta) - B \omega - \tau_L \right] \\
i_a &= \frac{1}{L_a} \left[ v_a - R i_a + K_m \omega \sin (N_r \theta) \right] \\
i_b &= \frac{1}{L_b} \left[ v_b - R i_b - K_m \omega \cos (N_r \theta) \right]
\end{align*}
\] (1)

Where \( v_a, v_b \) and \( i_a, i_b \) are the voltages and currents in phases A and B, respectively, \( \omega \) is the rotor (angular) velocity, \( \theta \) is the rotor (angular) position, \( B \) is the viscous friction coefficient, \( J \) is the inertia of the motor, \( K_m \) is the motor torque constant, \( R \) is the phase winding resistance, \( L \) is the phase winding inductance, and \( N_r \) is the number of rotor teeth. \( \tau_L \) is the load torque perturbation.

B. Conventional Microstepping Performance Analysis

If inputs \( v_{\text{ams}}^d, v_{\text{bms}}^d \) in conventional microstepping are given to the PMSM (1) with zero load torque \( \tau_L \). To move the position of the rotor to the desired position \( \theta_d \), the desired inputs are

\[
\begin{align*}
v_{\text{ams}}^d &= V_{\text{max}} \cos \left( N_r \theta_d \right), \\
v_{\text{bms}}^d &= V_{\text{max}} \sin \left( N_r \theta_d \right)
\end{align*}
\] (2)

Where \( \theta_d \) is the desired static position, and \( V_{\text{max}} \) is the amplitude of the microstepping input, then the states of the PMSM (1) locally asymptotically converge to one of the equilibrium points \( [\theta_d^d \quad 0 \quad i_{\text{ams}}^d \quad i_{\text{bms}}^d]^T \)

\[
\lim_{t \to \infty} \theta(t) = \theta_d, \quad \lim_{t \to \infty} \omega(t) = 0
\]
\[
\lim_{t \to \infty} i_a(t) = i_{\text{ams}}^d, \quad \lim_{t \to \infty} i_b(t) = i_{\text{bms}}^d
\] (3)

\[
i_{\text{ams}}^d = \frac{v_{\text{ams}}^d}{R} = \frac{V_{\text{max}}}{R} \cos \left( N_r \theta_d \right)
\]
\[
i_{\text{bms}}^d = \frac{v_{\text{bms}}^d}{R} = \frac{V_{\text{max}}}{R} \sin \left( N_r \theta_d \right).
\] (4)

Generally, various feedback controllers in the current loop are used to guarantee the desired currents using the PI controller, PI and feedforward controller, and nonlinear controller. In PMSM (1), the electrical dynamics is much faster than the mechanical dynamics. Therefore, a position tracking error appears despite that the feedback controller guarantees the desired currents \( i_{\text{ams}}^d \) and \( i_{\text{bms}}^d \) during the nonzero velocity period, as outlined in the following proposition. For the improvement of microstepping, PI feedback and feedforward controller is used.

III. CONTROLLER DESIGN

To overcome the limitation of conventional microstepping noted in Proposition 1, we design a controller consisting of three elements: nonlinear torque modulation, a current tracking controller, and a commutation scheme. Then, we compare the proposed methods with conventional microstepping based on both two-phase and DQ frames.

A. Nonlinear Torque Modulation via Backstepping

In the context of mechanical \((\theta, \omega)\) dynamics, the desired torque \( \tau_d \) can be regarded as an input as follows:

\[
\dot{\theta} = \omega
\]
\[
\dot{\omega} = \frac{1}{J} \left[ \tau_d - B \omega - \tau_L \right]
\] (5)

where

\[
\tau_d = -K_m i_a \sin (N_r \theta) + K_m i_b \cos (N_r \theta)
\]

The tracking errors of the mechanical dynamics are defined as

\[
\varepsilon_\theta = \theta_d - \dot{\theta}, \quad e_\omega = \omega_d - \omega
\] (6)

Where \( \theta_d \) is the desired dynamic position and \( \omega_d \) will be defined in the following lemma. The tracking error dynamics of the mechanical dynamics is given by

\[
\dot{\varepsilon}_\theta = \varepsilon_\omega - \varepsilon_\omega - \frac{1}{J} \left[ \tau_d - B \omega - \tau_L \right]
\]
\[
\dot{e}_\omega = \omega_d - \frac{1}{J} \tau_d - B \omega - \tau_L.
\] (7)
**Lemma 1:** Consider the tracking error dynamics (7). If the nonlinear torque modulation is designed by
\[
\dot{\omega}^* = \dot{\omega}^d + k_1 (\dot{\theta}^d - \dot{\theta})
\]
\[
t^d = k_2 (\dot{\omega}^* - \dot{\omega}) + (\dot{\theta}^d - \dot{\theta} + B_\omega + J\dot{\omega}^* + r_\tau)
\]  
(8)

Where \(k_1\) and \(k_2\) are positive constant and \(\omega^d = \dot{\theta}^d\) is the desired velocity, then the origin of the tracking error dynamics (7) is exponentially stable.

**Proof:** For stability analysis, \(V_{m1}\) is defined as
\[
V_{m1} = \frac{1}{2} e_{\theta}^2.
\]  
(9)

With \(\dot{\omega}^* = \dot{\omega}^d + k_1 (\dot{\theta}^d - \dot{\theta})\), \(V_{m1}\) becomes
\[
\dot{V}_{m1} = -k_1 e_{\theta} e_{\omega} + e_{\omega} e_{\omega}.
\]  
(10)

Then, to go one step ahead, \(V_{m2}\) is defined as follows:
\[
V_{m2} = \frac{1}{2} e_{\omega}^2 + \frac{1}{2} e_{\theta}^2.
\]  
(11)

The derivative of \(V_{m2}\) with respect to time is given by
\[
\dot{V}_{m2} = e_{\omega}(\dot{\omega}^* - \dot{\omega}) + e_{\omega}(J\dot{\omega}^* - t^d + B_\omega + r_\tau).
\]  
(12)

With the control input (8), \(V_{m2}\) becomes
\[
\dot{V}_{m2} = -k_1 e_{\theta}^2 - k_2 e_{\omega}^2 < 0.
\]  
(13)

Therefore, the origin of the tracking error dynamics (7) is exponentially stable.

**B. Nonlinear Current Tracking Controller**

The nonlinear torque modulation (8) is designed under the assumption that the desired torque \(t^d\) is the input in the mechanical dynamics and the electrical dynamics is neglected. Since the actual input is not the current but the voltage in PMSM, the torque \(r_\tau\) actually generated by the current is not the desired torque \(t^d\). Therefore, the nonlinear current tracking controller is designed to track the desired currents \(i^d_1, i^d_2\), which will be defined in the next section for generation of the desired torque (8). Decreases of current and phase lags appear in the electrical dynamics (\(i_a, i_b\)) due to the effects of back-emf and inductance during nonzero velocity periods. Therefore, the nonlinear controller is designed to guarantee the desired currents that will be designed in this section.

Let us define the tracking errors of the electrical dynamics as
\[
e_a = i^d_a - i_a, \quad e_b = i^d_b - i_b.
\]  
(14)

The tracking error dynamics of the electrical dynamics is given by
\[
\dot{e}_a = i^d_a - \frac{1}{L} [v_a - R_i a + K_m \omega \sin(N_\tau \theta)]
\]
\[
\dot{e}_b = i^d_b - \frac{1}{L} [v_b - R_i b - K_m \omega \cos(N_\tau \theta)].
\]  
(15)

**Lemma 2:** Consider the tracking error dynamics (15). If the nonlinear current tracking controller is given to the tracking error dynamics (15) as
\[
v_a = (R_i a - K_m \omega \sin(N_\tau \theta)) + L (i^d_a + k_3 e_a)
\]
\[
v_b = (R_i b + K_m \omega \cos(N_\tau \theta)) + L (i^d_b + k_3 e_b)
\]  
(16)

then the origin of the tracking error dynamics (15) is exponentially stable.

**Proof:** \(V_e\) is defined as
\[
V_e = \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2.
\]  
(17)

Differentiating \(V_e\) with respect to time yields
\[
\dot{V}_e = e_a \left( i^d_a - \frac{1}{L} (v_a - R_i a + K_m \omega \sin(N_\tau \theta)) \right)
\]
\[
+ e_b \left( i^d_b - \frac{1}{L} (v_b - R_i b - K_m \omega \cos(N_\tau \theta)) \right).
\]  
(18)

Substituting the control law (16) in (18) gives us
\[
\dot{V}_e = -k_3 \left( e_a^2 + e_b^2 \right) < 0.
\]  
(19)

Therefore, the origin of the tracking error dynamics (15) is exponentially stable. The nonlinear current tracking controller (16) guarantees that the currents exponentially converge to the desired currents.

**C. Commutation Scheme**

In this section, we will propose two commutation schemes to achieve FOC and FWC without DQ transformation. The desired currents \(i^d_a\) and \(i^d_b\) are defined based on microstepping as
\[
i^d_a = i^d \cos(N_\tau \theta_{ms}), \quad i^d_b = i^d \sin(N_\tau \theta_{ms}).
\]  
(20)

Where \(\theta_{ms}\), \(i^d\) will be defined in the following sections.

**Remark:** \(e^d\) and \((V_{max}/R)\) of \(i^d_a, i^d_b\) of conventional microstepping (4) are equivalent to \(\theta_{ms}\) and \(i^d\) of \(i^d_a, i^d_b\) of the proposed commutation scheme (20).
The DQ transformation [17] for the currents is defined as

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \begin{bmatrix}
\cos(N_r \theta) & \sin(N_r \theta) \\
-\sin(N_r \theta) & \cos(N_r \theta)
\end{bmatrix} \begin{bmatrix}
i_a \\
i_b
\end{bmatrix}
\]  \hspace{1cm} (21)

where \(i_d\) is the direct current and \(i_q\) is the quadrature current, respectively. Applying the DQ transformation to the desired currents (20), \(i_d^*, i_q^*\) becomes

\[
i_d^* = I^d \cos(N_r \theta_m - N_r \theta), \quad i_q^* = I^d \sin(N_r \theta_m - N_r \theta).
\]  \hspace{1cm} (22)

Since \(i_q = -I_a \sin(N_r \theta) + i_b \cos(N_r \theta)\) in PMSM (1), for generation of the desired torque (8) \(I^d\) is defined as

\[
I^d = \frac{\tau^d}{K_m \sin(N_r \theta_m - N_r \theta)}.
\]  \hspace{1cm} (23)

Then \(i_d^*, i_q^*\) are

\[
i_d^* = \frac{\tau^d \cos(N_r \theta_m - N_r \theta)}{K_m \sin(N_r \theta_m - N_r \theta)}, \quad i_q^* = \frac{\tau^d}{K_m}.
\]  \hspace{1cm} (24)

1) FOC: In order to maximize the torque, zero direct current should be maintained, i.e., \(i_d^* = 0\); that means \(\cos(N_r \theta_m - N_r \theta) = 0\). For achieving FOC, we define \(\theta_m\) as

\[
\theta_m = \frac{N_r \theta + 0.5 \pi}{N_r}.
\]  \hspace{1cm} (25)

Consequently, \(i_d^*, i_q^*\) become

\[
i_d^* = I^d \cos(N_r \theta + 0.5 \pi) = -\frac{\tau^d}{K_m} \sin(N_r \theta),
\]

\[
i_q^* = I^d \sin(N_r \theta + 0.5 \pi) = \frac{\tau^d}{K_m} \cos(N_r \theta).
\]  \hspace{1cm} (26)

Fig. 1. (a) Desired currents of conventional microstepping. (b) Desired currents of FOC.

Fig. 2. Nonlinear torque modulation (8) is used to compute the desired torque \(\tau^d\) for tracking the desired position. Then the commutation scheme (26) provides the desired current \(i_d^*\) and \(i_q^*\) for the desired torque. Finally, the nonlinear current tracking controller (16) provides the actual control voltages to guarantee the desired currents.
IV. ANALYSIS OF CLOSED-LOOP STABILITY

Since the proposed methods (8) and (16) are separately designed, the stability of the closed-loop should be proven. Let us define new errors as

\[ e_m = [e_\theta, e_v]^T, \quad e_e = [e_a, e_b]^T. \]  

(36)

The error dynamics of the closed-loop becomes

\[ \dot{e}_m = A_m e_m + B_m(\theta) e_e \]  

(37)

\[ \dot{e}_e = A_e e_e \]  

(38)

Where

\[ A_m = \begin{bmatrix} -k_1 & 1 \\ -1 & -k_2 \end{bmatrix} \]

\[ B_m(\theta) = \begin{bmatrix} 0 & K_m \sin(N_\theta) \\ 0 & K_m \cos(N_\theta) \end{bmatrix} \]

\[ A_e = \begin{bmatrix} -k_3 & 0 \\ 0 & -k_3 \end{bmatrix} \]

**Proof:** Let us choose \( k_1, k_2, \) and \( k_3 \) such that \( A_m \) and \( A_e \) are Hurwitz. \( V \) is defined as

\[ V = \frac{1}{2} e_m^T e_m. \]

The \( \dot{V} \) is given by

\[ \dot{V} = -e_m^T Q_m e_m + e_e^T B_m(\theta) e_e \]

Where

\[ Q_m = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \]

\[ B_m(\theta) \] is bounded.

If we define \( B_m(\theta)e_v \) as the input and \( e_m \) as the output in (37) and (40) can be rewritten as

\[ \dot{e}_m = \dot{V}_1 + e_m^T Q_m e_m > 0. \]

Equation (41) shows that the relationship between \( e_m \) and \( B_m(\theta)e_v \) is strictly output passive [19]. And \( \dot{e}_m = A_m e_m \) is zero-state observable since \( A_m \) is Hurwitz. Therefore, (37) is bounded input bounded output (BIBO) stable. Therefore, since the origin of (38) is exponentially stable, the origin of (37) is exponentially stable.

V. SIMULATIONS

Simulations and experiments were performed to evaluate the performance of the proposed controller. The desired position shown in Fig. 3 was used. During the constant velocity period, \( \omega_{\text{max}}^{d} = 8.75 \text{ rad/s} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L )</td>
<td>14.8 mH</td>
</tr>
<tr>
<td>( J )</td>
<td>( 8 \times 10^{-4} \text{ kg m}^2 )</td>
</tr>
<tr>
<td>( N_r )</td>
<td>50</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>0.04 N m rad</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>0.5</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>30</td>
</tr>
<tr>
<td>( V_{\text{max}} )</td>
<td>6.5</td>
</tr>
</tbody>
</table>

For the performance analysis of the proposed methods in the viewpoint of microstepping, the proposed methods FOC is compared with Lyapunov based control that is designed based on microstepping as follows:

\[ v_{d_{\text{DIS}}}^{d} = V_{\text{max}} \cos(N_\theta \theta^{d}) \]

\[ v_{p_{\text{DIS}}}^{d} = V_{\text{max}} \sin(N_\theta \theta^{d}) \]

\[ v_{a} = R_{\text{DIS}} - K_m \omega \sin(N_\theta \theta^{d}) + L \left( \dot{\theta}^{d} + k_3 \left( v_{d_{\text{DIS}}}^{d} - R_{\text{DIS}} \right) \right) \]

\[ v_{b} = R_{\text{DIS}} + K_m \omega \cos(N_\theta \theta^{d}) \]

\[ + L \left( \dot{\theta}^{d} + k_3 \left( v_{p_{\text{DIS}}}^{d} - R_{\text{DIS}} \right) \right). \]  

(42)

Fig. 3. Desired position \( \theta^d \).

Simulations were done using MATLAB/simulink. The proposed methods overcome the limitation associated with conventional microstepping by using torque modulation (8). Furthermore, the currents and voltages in the proposed methods were reduced compared with those of Lyapunov-based control, which were increased due to the constant amplitude of the desired current.
In FOC, zero direct current was maintained to maximize torque by the commutation (26), thus the currents were smaller than those of the Lyapunov-based control. Furthermore, during the constant velocity period, the quadrature currents were 0.47 A, and nearly identical to the value of 0.47 A obtained by (35).

![Currents and voltages of FOC. (a) Phase A voltage and Phase B voltage. (b) Phase A current and Phase B current](image)

VI. CONCLUSION

We presented microstepping with torque modulation to improve the position tracking performance of PMSMs and to establish the FOC and FWC methods.

The proposed method consists of nonlinear torque modulation, a commutation scheme, and a nonlinear current tracking controller. Nonlinear torque modulation was proposed to achieve the desired torque. The commutation schemes were developed to achieve FOC and FWC without DQ transformation. The nonlinear current tracking controller was designed to guarantee the desired current derived by the commutation scheme. It is shown that the commutation schemes are equivalent to microstepping, which has desired currents with time-varying amplitudes and electrical phase advance. Simulation and experimental results showed that the proposed method improved the position tracking performance of microstepping.

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