Abstract—State estimation, the core of the Energy Management System (EMS) is a prerequisite for operation of modern power grid. It changed its emergence with the introduction of high speed Phasor Measurement Unit (PMU) based Wide-Area Measurement Systems (WAMS) featured with synchronous sampling later leading to Dynamic State Estimation (DSE) due to slow update rate of SCADA systems. This paper deals with state estimation process, based on Kalman Filtering techniques, multi-machine power systems. Comparison of Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) algorithms is also done along with their verification under transient conditions. It is demonstrated that UKF is easier to implement and more accurate in estimation. Additionally, the parameter estimation for assumed ZIP load model is performed based on the Weighted Least Square (WLS) estimation method. A more accurate load modeling development and integration in DSE can be done as a future work.

Keywords— Dynamic State Estimation (DSE); Phasor Measurement Unit (PMU); Energy Management System (EMS); Extended Kalman Filter (EKF); Unscented Kalman Filter (UKF)

I. INTRODUCTION

The rapid growth and increasing complexity in recent years make the monitoring and control of power systems a very significant issue. Energy Management Systems (EMS) at the control centers is responsible for this task of monitoring and control of the system. The state estimator [9], which is the backbone of the energy management systems, provides an optimum real time data of the system state based on the available measurements on the assumed system model. The explanation stability of dynamic modeling[26]. J Wildes [27] analyzed static state estimation (SSE) and R Larson [28] developed a dynamic estimator (DE) model for a power system to evaluate the required states. T Koike [29], proposed a novel technique for detection of sudden variation in dynamic state estimator. L F Ahmed [30], developed measurement approach for the Tai-power system under dynamic loading conditions. J. K. Uhlmann [15], developed novel Unscented Transformation filter (UTF) for nonlinear transformation estimation to propagate covariance and mean information.

M. Zhao [31], proposed phasor measurement units (PMU) to investigate the state estimation problems in multi-area power system. H Renmu [32], presented measurement multi curve approach to identify parameters in composite dynamic load model. Z Huang [33] investigated feasible dynamic states via Extended Kalman Filter (EKF) approach. N R Shivakumar [34], briefly discussed various techniques to identify dynamic states in power systems. J Tate [35], developed PMU phasor angle measurements to detect outages in double line system. W Gao [16] analyzed the dynamic estimator model for a power system using Unscented Kalman Filter (UKF). E Ghahremani [13] extended Kalman filter (KF) to estimate dynamic states in power system with unknown inputs. M Gol [36] designed PMU to improve performance of SE and capabilities. G G Rigatos [37] introduced derivative free Kalman filter (DFKF) to SE based control for nonlinear dynamic estimator. D H Dini [38] introduced linear stochastic frequency estimators for tracking the unbalanced system conditions. Hao Zhu [39] proposed iterative optimization method to estimate the nonlinear states in power systems. Veerakumar Murugesan [40] suggested PMU buffer phasor measurements to detect bad data in SE power system. D Trivedi [41] investigates the EKF to track DE capability under varying noise content conditions. The real time power system may contain two types of errors: parametric errors and non parametric errors. Parametric errors are those errors whose occur in a power system due to the use of incorrect values of parameter such as incorrect value of resistance or reactance. In case of non parametric error approximate power system models are used instead of real one so the parametric error is taken in to account and it has been observed that Dynamic State Estimation is efficient for the presence of parametric error.

II. USE OF EKF AND UKF TECHNIQUES IN POWER SYSTEM DSE

A. Mathematical Modeling

The dynamic state estimation algorithms compute the variables of the state in non-linear algebraic equations representing the power system.
In the estimation process, the first step is the mathematical model identification for the time behavior of the power system. Making use of this identified mathematical model and the collected measurement data, DSE predicts the dynamic state vector ahead by one step. A dynamic system can generally be designed as a set of non-linear differential equations [12, 13]:

\[
\frac{dx}{dt} = f(x, u, w)
\]  

(1)

Where \( f(\cdot), x, u, w \) represents the system function, the state variables vector, algebraic variables vector and process (system) noise respectively. Eq. 1 in differential form is [12]:

\[
x_k = x_{k-1} + f(x_{k-1}, u_{k-1}, w_{k-1}) \Delta t
\]

(2)

Where \( k-1 \) is the instant of time index at present, \( k \) is the next instant of time index and \( \Delta t \) is the time step. The measurements at time step \( k \) can be represented as vector of non-linear functions \( h(\cdot) \) in terms of the state variables \( x \) and measurement noise \( v \) as below [12]:

\[
z_k = h(x_k, u_k)
\]  

(3)

The resulting error between the measured and calculated values is given by [12]

\[
e_k = z_k - h(x_k, u_k)
\]  

(4)

Since not all the dynamic variables of power systems can be measured directly, they need to be estimated and computed. The Kalman Filter techniques [3] execution can solve this problem as the Kalman Filter[5][7,8] has the ability of incorporating noise characteristics into the computations. The Extended Kalman Filter (EKF) [1][20] and Unscented Kalman Filter (UKF) [21][22] algorithms can be effectively used to estimate the multi-machine power system dynamics [4] which are the state variables in non-linear differential equations. The EKF and UKF algorithms are illustrated in the following sections.

### III. EXTENDED KALMAN FILTER (EKF) ALGORITHM

A two step prediction correction process, The Extended Kalman Filter (EKF) [25] algorithm can be summarized as follows [24]:

1. **The discrete time system equations are as follows:**

\[
x_k = f_{k-1}(x_{k-1}, u_{k-1}, w_{k-1})
\]

\[
y_k = h_k(x_k, u_k)
\]

\[
w_k \sim (0, Q_k)
\]

2. **Initialize the filter:**

\[
\hat{x}_0 = E(x_0)
\]

\[
P_0^+ = E[(x - \hat{x}_0)(x - \hat{x}_0)^T]
\]

(6)

where \( \hat{x}_0 \) represents the initial state and \( P_0^+ \) represents the initial state covariance matrix. The subscript + indicates the estimate is in a posteriori estimate. For \( k = 1,2, \ldots \) conduct the following operations:

3. **Compute the following partial derivative matrices at the current state estimate:**

\[
F_{k-1} = \frac{\partial f_{k-1}}{\partial x} \mid \hat{x}_{k-1}^-
\]

\[
L_{k-1} = \frac{\partial f_{k-1}}{\partial w} \mid \hat{x}_{k-1}^-
\]

(7)

4. **Perform the time update of [11] estimate and estimation-error of the covariance matrix:**

\[
P_k^- = F_{k-1}^+ P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T
\]

\[
x_k^+ = f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0)
\]

(8)

Where, the subscript - denotes that the estimate is in an a priori estimate.

5. **Perform the following partial derivative matrices at the state \( \hat{x}_k^+ \):**

\[
H_k = \left. \frac{\partial h_k}{\partial x} \right| \hat{x}_k^-
\]

\[
V_k = \left. \frac{\partial h_k}{\partial v} \right| \hat{x}_k^-
\]

(9)

6. **Perform state estimate measurement update and estimation covariance as given below:**

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}
\]

\[
\hat{x}_k = \hat{x}_k^+ + K_k [y_k - h_k(\hat{x}_k^+, 0)]
\]

\[
P_k^+ = (I - k_k H_k) P_k^-
\]

(10)

Where, \( K_k \) is the Kalman gain matrix, \( \hat{x}_k^+ \) is the state estimate and \( P_k^+ \) is the estimation error covariance matrix.

The Extended Kalman Filter (EKF) illustrated above is one of the most widely used estimation algorithm for estimating the non-measurable state variables of the non-linear systems.
The estimation of multi-machine dynamic variables including rotor speed and rotor angle can also be done successfully with valid performance in small and large disturbance conditions. The EKF method also linearizes all the non-linear transformations and then calculates Jacobian matrices. As known the linearization and Jacobian matrix calculation can lead to some serious drawbacks although the EKF method is computationally efficient in dynamic state estimation [14-19] [23]:

a. First, the linearization is reliable only if the higher order terms in Taylor expansion can be ignored. The linearized approximation would be extremely poor if this condition cannot hold, and as a result, the filter performance will be highly unstable if the time step is not set sufficiently small.

b. The linearization process can be applied only if the Jacobian matrices exist and it is impossible to perform linearization without it during the filtering process.

c. The other disadvantage of EKF is that the Jacobian matrix calculation is an absolute error-prone process as it includes many intricate partial derivations. The matrix calculation can be really CPU intensive as any error in coding or decoding process may lead to serious problems.

To overcome the above drawbacks of EKF method another method without including linearization and Jacobian matrix computation can be used instead of EKF in the dynamic studies of multi-machine power systems. This UKF based unscented transformation is more efficient, straightforward and easy to implement in the dynamic state estimation process.

IV. UNSCENTED KALMAN FILTER (UKF) ALGORITHM

It is a known fact that the linear approximations applied to the non-linear equations representing the dynamics can create reduced estimation performance and lower accuracy. Also there exists unstability in filtering as the high order terms in the Taylor series expansion are neglected. Now the Unscented Transform (UT) offers a good opportunity to overcome these limitations of linearization based EKF algorithm.

The statistical distribution of the state is propagated through the nonlinear equations which provide better approximation of state vector and covariance matrix [17]. The Unscented Transformation theory can be summarized as follows [17-19]:

Suppose x is an n-dimensional random variable with mean m and covariance Pxx. Then suppose another random variable y which is related to x through a non-linear function

\[ y = f(x) \]  \hspace{1cm} (11)

The key idea of unscented transformation is to obtain a deterministically chosen sigma points which capture exact mean and covariance of the original distribution of x. The sigma points are then used in calculating the mean \( \hat{y} \) and covariance \( P_{yy} \) of y.

\[ x^0 = m \]
\[ x^i = m + (\sqrt{(n+\lambda)P_{xx}^i})^{1/2} \quad i = 1, \ldots , n \]  \hspace{1cm} (12)
\[ x^{i+n} = m - (\sqrt{(n+\lambda)P_{xx}^i})^{1/2} \quad i = 1, \ldots , n \]

When \( (\sqrt{(n+\lambda)P_{xx}^i})^{1/2} \) is the ith row or column of matrix square root of the (n+λ) Pxx, λ can be defined as \( \lambda = \alpha^2(n+\lambda) - n \). It is suggested to use \( 10^{-4} \leq \alpha \leq 1 \) and \( k=3-n \) or \( k=0 \). The square root matrix can be approximated by \( P = AA^T \), where A is lower triangular matrix obtained from the Cholesky factorization of P [17].

In the next state, using non-linear function, the previously obtained sigma points can be transformed to get the transformed sigma points as below:

\[ x^i = f(x^i) \]  \hspace{1cm} (13)

Then the mean and covariance of y can be calculated using the previously calculated transformed sigma points as:

\[ y = \sum_{i=0}^{2n} W_m^i y^i \]
\[ P_m = \sum_{i=0}^{2n} W_c^i (x^i - y^i)(x^i - y^i)^T \]  \hspace{1cm} (14)

Where, the weights \( W_m^i \) and \( W_c^i \) are defined as:

\[ W_m^i = \frac{\lambda}{n+\lambda} \]
\[ W_c^i = \frac{\lambda}{n+\lambda} + (1 - \alpha^2 + \beta) \]
\[ W_l^i = W_c^i = \frac{1}{2(n+\lambda)} \]  \hspace{1cm} (15)

Where, the parameter \( \beta \) takes a value 2 which is typical for a Gaussian distribution.

The UKF algorithm consists of three main parts: calculation of sigma points, state prediction and state correction respectively.

The Unscented Kalman Filter (UKF) based on unscented transformation (UT) theory can be summarized as [24]:

1. The discrete time system equations are presented as follows:

\[ x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \]
\[ y_k = h(x_k, v_k) \]
\[ w_k \sim (0, Q_k) \]  \hspace{1cm} (16)
Where the system noise covariance matrix is represented by $Q_k$ and the measurement noise covariance matrix is represented by $R_k$.

2. **Initialize the filter:**

$$\hat{x}_0^+ = E(x_0)$$
$$P_0^+ = E[(x - \hat{x}_0^+)(x - \hat{x}_0^+)^T]$$

(17)

Where, $\hat{x}_0^+$ represents the initial state and $P_0^+$ represents the initial state covariance matrix. The subscript + indicates the estimate is in an a posteriori estimate.

3. **The following equations are used to propagate the state estimate and covariance from one measurement time to the next.**

- Firstly, to propagate from time step $k-1$ to $k$, the sigma points $\hat{x}_k^{(i)}$ are specified as per the formula:

$$\hat{x}_{k-1}^{(i)} = \hat{x}_{k-1}^+ + \hat{x}_{k-1}^{(i)} \quad i = 1, \ldots , 2n$$

$$\tilde{x}^{(i)} = (\sqrt{(n + \lambda)P_{k-1}^{+}})^T \quad i = 1, \ldots , n$$

(18)

$$\tilde{x}^{(n+i)} = -(\sqrt{(n + \lambda)P_{k-1}^{+}})^T \quad i = 1, \ldots , n$$

- To transform the sigma points into $\hat{x}_k^{(i)}$ vectors, use the known nonlinear system equation $f(.)$ as shown in Eq18 with the appropriate changes since our main nonlinear transformation is $f(.)$ rather than $h(.)$:  

$$\hat{x}_{k-1}^{(i)} \rightarrow \hat{x}_k^{(i)} \quad \hat{x}_k^{(i)} = f(\tilde{x}_{k-1}^{(i)}, u_k, t_k)$$

(19)

- Combine the $\hat{x}_k^{(i)}$ vectors to obtain the a priori state estimate at time $k$ by

$$\hat{x}_k^- = \frac{1}{2n} \sum_{i=1}^{2n} \hat{x}_k^{(i)}$$

(20)

- Estimate the a priori error covariance by adding $Q_k$ to take the process noise into account:

$$P_k^- = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{x}_k^{(i)} - \hat{x}_k^-)(\hat{x}_k^{(i)} - \hat{x}_k^-)^T + Q_{k-1}$$

(21)

4. **The time update equations are completed here and the measurement update equations need implementation in the final part of the UKF algorithm.**

- Choose sigma points $\tilde{x}_k^{(i)}$ with appropriate changes as the current best guess values are $\hat{x}_k^- \text{ and } P_k^- :$

$$\tilde{x}_k^{(i)} = \hat{x}_k^- + \tilde{x}_{k-1}^{(i)} \quad i = 1, \ldots , 2n$$

$$\tilde{x}^{(i)} = (\sqrt{(n + \lambda)P_{k-1}^-})^T \quad i = 1, \ldots , n$$

(22)

(b) Use the known $h(.)$ to transform the sigma points into $\tilde{y}_k^{(i)}$ vectors:

$$\tilde{y}_k^{(i)} = h(\hat{x}_k^{(i)}, t_k)$$

(23)

(c) Combine the $\tilde{y}_k^{(i)}$ vectors to obtain the predicted measurement at time $k$:

$$\tilde{y}_k = \frac{1}{2n} \sum_{i=1}^{2n} \tilde{y}_k^{(i)}$$

(24)

(d) Estimate the covariance of the predicted measurement by adding $R_k$ to the end of the equation in order to take the measurement noise into account:

$$P_y^- = \frac{1}{2n} \sum_{i=1}^{2n} (\tilde{y}_k^{(i)} - \tilde{y}_k)(\tilde{y}_k^{(i)} - \tilde{y}_k)^T + R_k$$

(25)

(e) Estimate the cross covariance between $\hat{x}_k$ and $\tilde{y}_k$:

$$P_{xy}^- = \frac{1}{2n} \sum_{i=1}^{2n} (\hat{x}_k^{(i)} - \hat{x}_k)(\tilde{y}_k^{(i)} - \tilde{y}_k)^T$$

(26)

(f) The state estimate measurement update can be performed using the normal Kalman filter equations:

$$K_k = P_{xy}^- P_y^-$$
$$\hat{x}_k^+ = \hat{x}_k^- + K_k(\tilde{y}_k - \tilde{y}_k)$$

(27)

$$P_k^+ = P_k^- + (K_k P_y^- K_k^T)$$

where $K_k$ is the Kalman gain matrix, $\hat{x}_k^+$ is the state estimate and $P_k^+$ is the estimation error covariance matrix.

V. **Results**

In this section, the dynamic variables, the generator rotor speed $\omega$ and the generator rotor angle $\delta$ are estimated by using both Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) techniques. The estimation process is simulated on various multi-machine test power systems [6]. During these simulations, different scenarios of bus fault, sudden load change, line switch were considered to test the estimation performance of both the techniques. These are applied to the multi-machine test systems in a definite time interval to observe the change in the behavior of the dynamic state variables.
The EKF and UKF algorithms are applied to estimate the actual behavior of the dynamic variables under various transient conditions. Also the simulation and filtering pacifications are definitely kept same for both algorithms. The bus real power injections, bus voltage magnitudes, bus voltage angles and the reactive power injections are used as measurement vector during the estimation process.

The dynamic state vector $x$ and the measurement vector $z$ can be shown as below:

$$x = \begin{bmatrix}
\omega_1 \\
\omega_2 \\
\vdots \\
\omega_n \\
\delta_1 \\
\delta_2 \\
\vdots \\
\delta_n 
\end{bmatrix}$$

$$z = \begin{bmatrix}
P_{G1} \\
P_{G2} \\
\vdots \\
P_{Gn} \\
Q_{G1} \\
Q_{G2} \\
\vdots \\
Q_{ Gn} \\
|V_{1}| \\
|V_{2}| \\
\vdots \\
|V_{n}| \\
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_n
\end{bmatrix}$$

(28)

Where, $n$ represents the number of generators and $s$ represents the number of buses in the power system.

The general Kalman filtering process principle applied during the dynamic estimation process of the test power systems can be summarized on the Figure 3.1 below.

![Figure 1: Kalman Filter is simply a two-step prediction-update process](image)

As summarized in Figure 1, the state vector for the instant of time $k + 1$ is predicted using the measurements and the state estimates at time instant $k$ along with the mathematical model of the test power system. A random Gaussian noise with zero mean and standard deviation, $\sigma=10^{-2}$ is assumed for both system noise and measurement noise during simulations. This indirectly means that the diagonal elements of the matrices system noise covariance, $Q$ and measurement noise covariance, $R$ are set to $\sigma^2=10^{4}$.

The initial values of the state estimate vector $\hat{x}_k$ are chosen arbitrarily to be same for both EKF and UKF. Simulations are carried out using all numerical integration methods of Euler method, Second Order Runge Kutta and Fourth Order Runge Kutta method. However, Fourth Order Runge Kutta method results are presented as it performs better and more accurate transient stability solution when compared to rest of the test systems. The base is assumed to be $S_{base} = 100$ MVA and the system frequency is assumed as $60$ Hz for all simulations and the test systems.

The comparison of EKF and UKF performances is made based on performance indices of:

**Estimation Error ($\xi$):**

The estimation error for the time instant $k$ is calculated by using the following formula [32]:

$$\xi_k = \frac{1}{2n} \sum_{i=1}^{2n} |x_k^i - \hat{x}_k^i|$$

(29)

Where, $2n$ is the number of states ($n$ rotor speed and $n$ rotor angle), $x$ is the actual state vector calculated at the end of the transient stability analysis and $\hat{x}$ represents the estimated state vector during the Kalman filtering process.

The overall estimation error is defined using the formula of mean of the estimation error vector including the estimation error in each computation step:

$$\xi = \frac{1}{K_{\text{max}}} \sum_{k=1}^{K_{\text{max}}} \xi_k$$

(30)

The estimation error of the generator rotor speed $\omega$ variables and the generator rotor angle $\delta$ variables is calculated using the formula separately as shown:

$$\xi_{\omega} = \frac{1}{K_{\text{max}}} \sum_{k=1}^{K_{\text{max}}} \left[ \frac{1}{n} \sum_{i=1}^{n} |\omega_k^i - \hat{\omega}_k^i| \right]$$

$$\xi_{\delta} = \frac{1}{K_{\text{max}}} \sum_{k=1}^{K_{\text{max}}} \left[ \frac{1}{n} \sum_{i=1}^{n} |\delta_k^i - \hat{\delta}_k^i| \right]$$

(31)

Where, $n$ is the number of $\omega$ and $\delta$ in the state vector.

The EKF and UKF performances are presented in the following section including the plots of the dynamic states and the performance indices for different test power systems.

**IEEE 5-Generator 14-Bus Power System**

The one-line diagram of the IEEE 5-generator 14-bus test power system is shown on Figure 2. The generator dynamic data is given in Table 1. The time step is set as $\Delta t = 0.02$s. The generator rotor speed $\omega$ and generator relative rotor angle $\delta$ for three generators are estimated considering the transient case described below:

**Transient Case:** Line 2-4 is removed at $t = 2$s.
**Generator 5**: The relative rotor angle $\delta_{5-1}$ is estimated by using EKF as shown on Figure 5 and UKF as shown on Figure 6.

**Table 1**: Generator dynamic data of the 5-generator 14-bus power system

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$G_1$</th>
<th>$G_2$</th>
<th>$G_3$</th>
<th>$G_4$</th>
<th>$G_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_d$</td>
<td>0.299(p.u)</td>
<td>0.185(p.u)</td>
<td>0.19(p.u)</td>
<td>0.23(p.u)</td>
<td>0.23(p.u)</td>
</tr>
<tr>
<td>$H$</td>
<td>3.13(s)</td>
<td>6.54(s)</td>
<td>6.54(s)</td>
<td>5.06(s)</td>
<td>5.06(s)</td>
</tr>
<tr>
<td>$D$</td>
<td>2(p.u)</td>
<td>2(p.u)</td>
<td>2(p.u)</td>
<td>2(p.u)</td>
<td>2(p.u)</td>
</tr>
</tbody>
</table>

**Generator 5**: The rotor speed $\omega_5$ is estimated by using EKF as shown on Figure 3 and UKF as shown on Figure 4.

**Table 2**: Performance indices of EKF and UKF for the 5-generator 14-bus power system

<table>
<thead>
<tr>
<th>Performance Index</th>
<th>EKF</th>
<th>UKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0.0119</td>
<td>8.5347e-04</td>
</tr>
<tr>
<td>$\xi_{\omega}$</td>
<td>3.0874e-04</td>
<td>3.5764e-05</td>
</tr>
<tr>
<td>$\xi_{\delta}$</td>
<td>0.0235</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

**VI. CONCLUSION**

The Dynamic State Estimation (DSE) plays a major role in real monitoring and control of large scale power grids by calculating the state by considering the time varying behavior of the system.
The introduction of the Phasor Measurement Units (PMUs) into power systems. The Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) techniques in the application of dynamic state estimation for multi-machine power systems are presented in this thesis. As seen from the results, the UKF algorithm is much superior to the EKF as it is more efficient, robust, easy to implement and has lower computational demand. The estimation error values of EKF and UKF algorithms illustrates that UKF gives more accurate performance under different transient conditions.

REFERENCES


