XFEM, Crack Growth Examination Of Cryo-Rolled (CR) 6082 Al Alloys

Vineet Kumar¹, Indra Vir Singh², Bhanu Mishra³

Abstract -In the present work, the effect of cryorolling rolling have been experimentally examined on the tensile strength, impact toughness and fracture energy of 6082 Al alloy. The crack growth simulations are performed using extended finite element method (XFEM) for cryorolled (CR) Al alloy. The alloy is CR up to 40% and 90% thickness reductions. An improvement in mechanical properties has been observed for CR samples through Charpy and impact testing. The fracture energy obtained by Charpy test is used as crack growth criterion for elasto-plastic crack growth simulations.

Index Terms— Cryorolling; XFEM simulation; energy criterion

I. INTRODUCTION

The 6XXX series alloys are aluminium/magnesium alloys and possesses highest strength among other series alloy. These are broadly used in fabrication of parts and structural components. This series contains exceptional mechanical properties such as low density, moderate strength, heat treatable, high corrosion resistance, ductility, toughness and good fatigue resistance. The unique feature of this alloy is great extrudability due to which it is possible produce single shapes easily such as complex architectural shape as well as metal design shapes. This alloy characteristic is a mainly important for architectural work and members where stiffness is critically important. The major use of 6082 Al alloy is in automotive industry. In previous literature [1-3] shows an improved tensile, hardness mechanical properties in ultra-fine-grained (UFG) shape of Cu, Ni, Al alloys. There is limited literature on UFG Al 6000 series alloys which is processed by cryorolling technique. The present work experimentally examined the properties of tensile and impact samples of the UFG rolled 6082 Al alloy. Afterward XFEM crack growth simulation is carried out.

As usual cryogenic rolling process (rolling at liquid nitrogen temperature) is preformed and ultra fine grains are produced at this temperature. Since the dynamic recovery is reduced by cryorolling, the CR Al 6082 alloy shows high fraction of high angle grain boundaries and high amount of dislocation density. The experimental work shows that the tensile and impact toughness properties are improved more in case of CR Al 6082 alloy as compared to its bulk alloys.

The crack growth XFEM numerical simulations are carried out by Abaqus 6.11 software. During Charpy impact test the fracture energy absorbed by the bulk, CR specimens is used as the crack growth criterion for XFEM simulations.

Explicit crack propagation criterion based on Griffith energy theory is used for elastic-plastic ductile fracture simulations. The boundary changes occur due to crack growth and the non-linearity of the material arises due to ductile fracture. Also, in fracture zone area, the plastic deformation and crack surface partition is distinguished as a result of the nucleation, growth and coalescence of micro-voids [4-6]. The crack growth simulations are performed using the theory of plastic deformation under quasi-static Mode-I loading [7-8]. A brief description of XFEM and energy criterion for the crack growth is given in section II, respectively.

II. EXTENDED FINITE ELEMENT METHOD

A. Comparison with FEM

The extended finite element method (XFEM) has proved to be a competent mathematical tool that enables a local enrichment of approximation spaces. The enrichment is recognized by the partition of unity concept [9]. This concept gives approximate solution having well-known non smooth characteristics in small parts with computational domain i.e discontinuities and singularities near the crack tip.

For crack growth simulation problems standard numerical methods such as finite element or finite volume often exhibit poor accuracy. The XFEM offers significant advantages by enabling optimal convergence for these applications.

[10-11] first time used the combination of the asymptotic fields of linear elastic fracture mechanics as well as a discontinuous function in a typical finite element approximation. The extended finite element Method can be used in a variety of fracture mechanics problems, including non-planar crack growth in three dimensions, cohesive crack growth, and dynamic crack growth.
For fracture mechanics problems having the crack can be solve by the finite element method (FEM) techniques which is most powerful tools for numerical solutions while extended finite element method (XFEM) allows you to calculate the crack propagation without remeshing finite elements.

The advantage over FEM defined as; firstly, in FEM modeling for crack geometry a re-meshing is required. However, XFEM modeling of crack propagation required enrichment functions due to which regular mesh is used. Secondly, in FEM, singular elements are used to model the asymptotic crack tip displacement fields, while in case of XFEM, the same can be modeled by employing branch function written for the asymptotic crack tip geometries. Thirdly, In FEM, as crack grows, a recreate of mesh is necessary around the crack tip, which is fairly expensive while in XFEM, no such exercise is essential. In a nutshell, XFEM is the PU enriched finite element method. In other words, XFEM is the PU enriched finite element method. Level Set Method (LSM) has been effectively conjugated with XFEM for defining as well as tracking the geometry of cracks. The X-FEM is superior to classical FEM due to accurate numerical results on the other hand; the convergence rate is not most favorable with respect to mesh parameter. This rate is worse compared with traditional FEM.

- Mesh creation from the geometry including discontinuities but in FEM excluding these discontinuities.
- Standard finite element method is not well suited for solid-solid, solid-fluid and fluid-fluid dynamics interfaces due their interpolation character. The shape functions of the standard finite element are piecewise polynomials and strong and weak discontinuities can be incorporated only along element edges. For the crack results become very sensitive with respect to the orientation of the mesh and also with respect to chosen size and shape of the elements. XFEM can be handle arbitrary strong and weak discontinuities.
- Extended finite able to naturally reproduce the challenging feature associated with the problem of interest: the discontinuity, singularity, boundary layer.
- Extended finite element methods mimes the need to mesh and remesh the discontinuity surfaces, thus facilitate the computational costs and projection errors associated with conventional finite element methods, at the cost of restricting the discontinuities to mesh edges.

Consider an n-dimensional domain 2 < n which is discretized by n elements numbered from 1 to n. I notation show the set of all nodes in the domain. The approximation for the vector function u with the partition of unity enrichment is of the general form.

\[ u^b(X) = \sum_{i=1}^{n} N_i(X) \sum_{a=1}^{n} w_a x_i^a \]  

Where represents the element shape function forms a partition of unity. By using the notion of unity, the standard approximation is enriched with additional functions. In particular instance of 2-D crack modeling, the enriched displacement trial and test approximation is written as,

\[ u^b(X) = \sum_{i=1}^{N} N_i(X) \left[ u_i + H(X) a_i + \sum_{a=1}^{M} \Phi_a(X) b_i^a \right] \]  

= Nodal displacement vector for finite element solution
= Nodal enriched additional degree of freedom vector for Heaviside (discontinuous) function
= Nodal enriched degree of freedom vector associated with the elastic asymptotic crack tip functions
N = Set of all nodes in the mesh
N_i = The support of the nodal shape function is fully cut into two disjoint pieces by the crack
H(X) = Heaviside jump function (discontinuous function around the crack surface), constant on crack side i.e. +1 on single side.

The enrichment function for the crack tip is done by addition supplementary functions in to displacement approximation as:

\[ \Phi_{-\theta}(X) = \frac{1}{2} \cos \theta, \frac{1}{2} \sin \theta, \frac{1}{2} \cos \frac{1}{2} \sin \theta, \frac{1}{2} \cos \frac{1}{2} \sin \theta \]  

Where, r and \( \theta \) define as local crack tip parameters at the estimation point. For this material X and n belong to spatial coordinates and Ramberg-Osgood constant.

For crack modeling in XFEM, two kinds of enrichment functions are defined, first function is Heaviside function, H(X) and second function is crack-tip functions. Eq. (1) turns into traditional finite element approximation when no enrichment is present. Figure 1 gives you an idea about enriched nodes that are used for crack modeling.
Fracture energy that is calculated through Charpy impact test is used as a criterion for crack propagation under quasi-static loading. Charpy impact energy (Total fracture energy) is a combination of fracture initiation energy and fracture propagation energy as per Eq. (4).

\[ U = U_i + U_p \]  \hspace{1cm} (4)

Where, \( U \) = total fracture energy; \( U_i \) = fracture initiation energy; and \( U_p \) is fracture energy during crack propagation.

The basic principle for energy conservation relating to crack expansion per unit area [12] is founded on Griffith fracture mechanics principle, scientifically confirmed as [13],

\[ T_r = \frac{\partial}{\partial A} (W_p - U_e - U_p - E_s) \]  \hspace{1cm} (5)

Where, \( T_r \) = total necessary energy for crack propagation

\( U_e \) = elastic strain energy of the system
\( U_p \) = plastic strain energy of the system
\( W_p \) = work done by the externally applied loadings
\( E_s \) = Surface dissipated energy and \( A \) is the total crack surface area [14-16].

For crack growth the strain energy release rate (\( G \)) which is measured as a driving force is the result of first two terms in Eq. (5).

\[ G = \frac{\partial(W_p - U_e)}{\partial A} \]  \hspace{1cm} (6)

The plastic energy dissipation rate and the energy dissipation rate which are results of crack surface separation are represented by last two terms of Eq. (5). The Energy dissipation rate ‘\( R \)’ which is defined as resistance to crack growth is represented by these last two terms.

\[ R = \frac{\partial(U_p + U_i)}{\partial A} \]  \hspace{1cm} (7)

In finite element method, the external work ‘\( P \)’ and the total strain energy ‘\( U \)’ of the system is represented as:

\[ U = U_e + U_p = \int V WdV \]  \hspace{1cm} (8)

Where, ‘\( V \)’ indicates the system volume, ‘\( u \)’ is nodal displacement vector, ‘\( F \)’ refers nodal corresponding force vector and ‘\( W \)’ is total strain energy density. Eq. (9) shows the volume integration which is carried out through element by element way. In all elements, first we calculate strain energy for each element.

### III. EXPERIMENTAL TESTING

Table 1 give an idea about the tensile properties of bulk, cryorolled 6082 Al alloy for different thickness reductions i.e. 40% and 90%. The yield strength of CR samples is increased from 260.3 MPa to 414.4 MPa (59.2% increase) and 260.3 MPa to 612 MPa (135.11% increase) and the ultimate tensile strength of CR samples is increased from 340 MPa to 423.3 MPa (24.5% increase) and 340 MPa to 645 MPa (89.7% increase) for 40% and 90% thickness reductions respectively as shown in Table-I CR Al alloys show higher yield strength and ultimate tensile strength due to reduction in the grain size.

The impact toughness properties of bulk, CR 6082 Al alloy are measured at thickness reductions of 40% and 90%. The impact energy of bulk Al alloy is found to be 12J whereas in case of CR alloy, it increased to 14J (16.7% increase), and 25J (108.3% increase) for 40% and 90% thickness reductions respectively.

### IV. XFEM SIMULATIONS

For XFEM simulation elastic- plastic nature are assumed for bulk alloy and CR Al alloy. The simulation is performed through ABAQUS using the plain strain condition for the current work. The working out of the strain energy release rate, plastic dissipation and the stress allocation in front of the crack tip, the improved fracture energies of cryorolled alloy are observed as judge against the bulk alloy. The maximum hoop stress criterion is followed for the crack growth direction.

The ultra-fine grain properties are obtained by the cryorolling technique after subsequent thickness reduction of the bulk alloy.

Using Ramberg-Osgood relationship [17-18] for the crack propagation mode-I loading condition with mesh independent criteria under deformation plasticity theory is used. Using the deformation plasticity concept the material behavior is modeled by a polynomial popularly known as Ramberg-Osgood relation. This relation implies that plastic strain is present at very low stress levels and it is mainly useful for metal that harden with plastic deformation. The material follows only one function as shown in Eq. (10).


\[ E\varepsilon = \sigma + \alpha \left( \frac{\sigma}{\sigma_0} \right)^{n-1} \sigma \quad (10) \]

Where, \( \sigma = \) stress, \( \varepsilon = \) strain, \( E = \) Young’s modulus of elasticity, \( \alpha = \) yield offset (0.2\%), and \( n (>1) \) is the hardening exponent that depends upon material being considered. First part of Eq. (10) shows the elastic strain energy part and second part shows plastic energy part. For fracture mechanics problems with small strain, this model gives the fully plastic solutions. The strain energy density required for evaluating strain energy release in this model is computed as:

\[ W = \int \sigma . d\varepsilon \quad (11) \]

The stress tensor considered for the material model gives the constitutive routines for the final kinematics solution at every integration point during the analysis. The material model is having non-linear relationship for the stress solution as described in Eq. (10), this Eq. gives the value of \( \sigma \) to the corresponding strain value of \( \varepsilon \). Due to non-linearity of Eq. (10) Newton-Raphson method is used to find the value of \( \sigma \) \([19\text{-}20]\). A correction factor \( c_\sigma \) to \( \sigma \) in this method is given as:

\[ \left( 1 + n\alpha \left( \frac{\sigma}{\sigma_0} \right)^{n-1} \right) c_\sigma = E\varepsilon - \sigma - \alpha \left( \frac{\sigma}{\sigma_0} \right) \sigma \quad (12) \]

The value of material stiffness matrix is specified by:

\[ \sigma = \sigma + c_\sigma \quad (13) \]

Here if and if

The current material stiffness matrix value for this case is specified as,

\[ \frac{d\sigma}{d\varepsilon} = \frac{E}{1 + n\alpha(\sigma/\sigma_0)^{n-1}} \quad (14) \]

A comparison between deformation plasticity and other elasto-plasticity models are shown below. For metal plasticity calculations an isotropic elasto-plasticity model is usually used (rate-dependent or rate-independent model). The strain rate in terms of elastic and plastic strain can be expressed as,

\[ \varepsilon = \varepsilon^{el} + \varepsilon^{pl} \quad (15) \]

The elastic behavior of the model is supposed as linear and isotropic and, that can be expressed as two temperature dependent material parameters.

These parameters are chosen as bulk modulus (K) and shear modulus (G), which can be expressed in terms of Young’s modulus ‘E’ and Poisson’s ration ‘\( \nu \)’ as,

\[ K = \frac{E}{3(1 - 2\nu)} \quad \text{and} \quad G = \frac{E}{2(1 + \nu)} \quad (16) \]

An increase in plastic flow is determined from the value of \( q \) which depends on the elastic response of the material. The value of ‘\( q \)’ can be calculated from Eq. (17), this equation is same for both rate-independent and integrated rate-dependent model.

\[ q = \sigma \varepsilon^{pl} \quad (17) \]

Where, \( \varepsilon^{pl} = \) plastic strain, (Abaqus Theory Manual, 2010).

Johnson-Cook plasticity model theory is best suited for those metal models having high-strain-rate deformation. Particularly Johnson-Cook model is a type of Mises plasticity that involves hardening law and rate dependence. This model uses a surface which is known as Mises yield surface with associated flow. A meticulous form of isotropic hardening is recognized as Johnson-Cook hardening, in which static yield stress ‘\( \sigma_y \)’ is assumed as,

\[ \sigma_y = \left[ A + B \left( \varepsilon^{pl} \right)^n \right] \quad (18) \]

Where A, B and n are material constants to be calculated experimentally.

<table>
<thead>
<tr>
<th>Form of Alloy</th>
<th>Yield strength, ( \sigma_y \text{ (MPa)} )</th>
<th>Tensile strength, ( \sigma_{uts} \text{ (MPa)} )</th>
<th>% Elongation</th>
</tr>
</thead>
<tbody>
<tr>
<td>40% rolled</td>
<td>414.4</td>
<td>423.3</td>
<td>16.0</td>
</tr>
<tr>
<td>90% rolled</td>
<td>612.0</td>
<td>645</td>
<td>14</td>
</tr>
<tr>
<td>Bulk</td>
<td>260.3</td>
<td>340.0</td>
<td>12.0</td>
</tr>
</tbody>
</table>

The single edge crack and center crack are considered for understanding crack growth mechanism and the present work is founded on the concept of grain enhancement into the ultra-fine-grain by cryorolling.
CR lead to the improvement in the yield strength of the material due to work hardening of the material and this phenomenon is best described by the deformation plasticity. It uses one polynomial function to model the behavior of the material from elastic to plastic range by taking into account the yield strength as well as hardening exponent of the material.

**Case I: FINITE SIZE PLATE WITH AN EDGE CRACK**

A plate having dimensions 80mm×40mm×10mm [height (h)×width(b)×thickness(t)] with a 5 mm crack length (a) is modeled and subjected to displacement loading as shown in Figure-1. The plate is meshed with quadrilateral bilinear elements. The lower edge of the plate is fixed and displacement (d) of 0.35 mm is applied on the edge opposite to the fixed edge. In the present work the elastic-plastic ductile fracture behavior of bulk and ultra-fine grain 6082 Al alloy is studied. Due to transition from tensile to shear governed crack growth the crack would propagate in the straight direction. Figure-2 shows the single edge crack in mesh form. The crack growth model of cryorolled (CR) at 90% thickness reduction is shown in Figure-3.

Figure-4 shows that the plastic dissipation in case of CR alloy is lower as compared to bulk alloy under the action of same displacement loading due to effect of work hardening for the whole model (Plastic dissipation of Al alloy is observed as CR 90% < CR 40% < Bulk alloy). Figure-5 observed that strain energy in CR 6082 Al alloy is more as compared to its bulk alloy at the fracture process region near the crack tip. Figure-6 shows that external work is higher in case of CR 90% of 6082 Al alloy in comparison to its bulk alloy under the action of same displacement loading.

![Fig. 1. Single edge crack](image1.png)

![Fig. 2. Bulk alloy in Single edge crack shape. (mesh form)](image2.png)
Case II: FINITE SIZE PLATE WITH A CENTER CRACK

A plate having dimensions 80mm×40mm×10mm [height(h)×width(b)×thickness(t)] with a 5 mm crack length (a) at the centre is modeled and subjected to displacement loading as shown in Figure-7. The lower edge of the plate is fixed and displacement (d) of 0.35 mm is applied on the edge opposite to the fixed edge. For the plate with centre crack plastic dissipation, and external work and strain energy release with time are plotted in Figure-8-10 for bulk and CR 6082 Al alloys.
The current work shows the improvement in tensile properties and impact toughness in CR 6082 Al alloy than its bulk alloy. The behavior of crack growth is studied by using XFEM module in Abaqus software. The above results show that XFEM simulation judgment is reasonably efficient and versatile for solving fracture mechanics problems which are complex.

The improvement in fracture properties of CR 6082 Al alloys are shown by the plots of plastic dissipation, strain energy release rate and the external work through the simulation of crack propagation. Both the cases discussed above shows that under similar geometric, boundary and loading conditions the crack growth in UFG form of the alloy is lesser. Further in case of UFG form of alloy there is an improvement in the plastic region near the crack tip. Concluding both cases, the fracture properties of 6082 Al alloy are improved because of effective grain refinement in cryo-rolling.
REFERENCES


AUTHOR’S PROFILE

Mr. Vineet Kumar is a research scholar in department of Mechanical and Industrial engineering IIT Roorkee. He has done B. Tech and M. Tech, from GBPUAT University of Agril. & Technology, Pantnagar, Uttarakhand. He has attended the many national and international conferences.

Dr. Indra Vir Singh is working as an associate Prof. in the department of Mechanical Engineering in IIT Roorkee. He has done B. Tech from AMU Aligarh, M. Tech form IIT Delhi and Ph. D, from BITS Pilani. He has worked in the field of research work: Fatigue, fracture, XFEM, Iso-geometric and FGM composite materials. He has published many papers in reputed national and international journal. He has organized many short term courses in mechanical department

Dr. Bhanu Mishra is working as a Prof. in the department of Mechanical Engineering in IIT Roorkee He has worked in the field of research work: FEM, X-FEM, Mesh-free Methods, Iso-geometric Analysis, Nonlinear and Multi-scale Simulations, Fracture, Fatigue, Damage Mechanics, FGMs, Nano-Micro-Macro Comosites. He has done B. Tech from BHU Banaras, M. Tech from IIT Kanpur and Ph. D, BHU from Banaras. He has published many papers in reputed national and international journal. He has supervised many M. Tech. and Ph. D thesis. Many projects are handled during his teaching work. He has organized international conference in the mechanical department.

Vineet Kumar, Mechanical & Industrial Engineering Department, IIT Roorkee, Roorkee, India, Phone/ Mobile No.9410480876, (e-mail: vineetrose@gmail.com).

Indra Vir Singh, , Mechanical & Industrial Engineering Department, IIT Roorkee, Roorkee, India, Phone/ Mobile No.9410480876, (e-mail: ivsingh@gmail.com).

Third Author name, His Department Name, University/ College/ Organization Name, City Name, Country Name, Phone/ Mobile No., (e-mail: bhanuafme@gmail.com).