Thermal Analysis of Cross Flow Heat Exchangers

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Abstract: The objective of this study is to develop a steady state performance model for cross flow heat exchangers. Both sensible heat transfer and phase change have been considered in this research work. Various flow configurations such as parallel flow, counter flow and neither parallel nor counter flow have been analyzed. Effectiveness-NTU approach combined with energy balance is employed to develop a set of generalized equations for a given cross flow heat exchanger. The generalized equations expressed in matrix form can be readily solved to ascertain the overall and the intermediate thermal performance of the heat exchanger. Prior to building a prototype, this work shall help the engineers to determine the thermal performance of the heat exchanger and thus shall help in developing the most cost efficient and an optimum heat exchanger for a given application.

KEYWORDS - Steady state heat exchanger performance; Matrix approach; Cross flow heat exchanger

1. INTRODUCTION

This work is a direct extension of the work presented by Silaipillayarputhur and Idem [1]. Matrix approach presented by [1] have been further extended to study the sensible performance and phase change in cross flow heat exchangers. The generalized equations resulting from the application of energy balance and effectiveness – NTU approach have been presented in the matrix form. In sensible heat transfer, for the overall heat exchanger, the commonly encountered situation is that the tube side fluid is the maximum capacity rate fluid. Since all the heat exchangers are not configured this way, the generalized equations have been developed by considering the tube side fluid as the maximum and also as the minimum capacity rate fluid. The developed matrix equations can be readily solved to yield the intermediate and the overall thermal performance of the heat exchanger. Recognize that NTU encompasses materials of construction, flow circuiting, thermal and flow conditions, heat exchanger design and fouling.

Therefore by employing the concept of NTU, the developed generalized equations can be universally applied for any cross flow heat exchanger having the prescribed flow circuiting. Additionally, the availability of intermediate thermal conditions shall help in optimizing the heat exchanger during the design phase and shall also help in understanding the thermal stresses on various components of the heat exchanger. It must be noted that the simple applications of the matrix approach can be used as “building blocks” to develop performance models of heat exchangers with more complex flow circuiting.

There are a great number of references available in the literature pertaining to heat exchanger performance modelling. Silaipillayarputhur and Idem [1] proposed matrix approach to study the steady state sensible performance of multi-row multi-pass cross flow tubular heat exchanger. The matrix approach proposed by [1] uses the concepts of local effectiveness, energy balance, and NTU applied to every pass/row in the cross flow heat exchanger to predict thermal performance. Also, matrix method can predict the total effectiveness of assemblies of heat exchangers. Kays and London [2] described the $\varepsilon$ – NTU method for both design and performance calculations of relatively simple heat exchangers. Domingos [3] presented a general method of calculating overall performance and intermediate temperatures of complex crossflow heat exchangers using the concept of effectiveness and a local energy balance. The method did not explicitly employ the concept of NTU in the analysis, and therefore is not well-suited to design calculations. Pignotti & Shah [4, 5] discussed the tools developed previously (such as Domingos’ method, the Pignotti chain rule, etc.) to determine the $\varepsilon$ – NTU relationship for highly complex heat exchanger flow arrangements, and examined very complex heat exchanger flow arrangements and related them to simple forms for which either a solution existed or an approximate solution could be derived.
Pignotti and Cordeo [6, 7] developed a procedure to obtain analytical expressions for the mean temperature difference in cross flow heat exchangers and presented graphs for the mean temperature difference in typical air cooler configurations. Pignotti [8] considered six different configurations of a series assembly of two divided-flow heat exchangers and developed effectiveness relationships as a function of the heat capacity rate ratio, and the partial effectiveness of the components. Sekulic, Shah, and Pignotti [9] presented a review of the solution methods for determining the $\epsilon$ -- NTU relationships for exchangers with complex flow arrangements.

Chen and Hsieh [10] presented a simple and a systematic procedure to determine the effectiveness and exit temperatures of complex assemblies of identical heat exchangers. Stevens et al. [11] used a numerical integration procedure to obtain the mean temperature difference data in one-, two-, and three-pass crossflow heat exchangers. DiGiovanni and Webb [12] studied the uncertainty in effectiveness-NTU calculations for crossflow heat exchangers assuming that in many practical flow situations, a special condition could exist where the fluid was neither perfectly “mixed” nor perfectly “un-mixed”. Baclic [13] provided a simplified formula for calculating the effectiveness of crossflow heat exchangers. Navarro and Gomez [14, 15, 16] presented a new methodology for steady state crossflow heat exchanger thermal performance calculations. Their approach was characterized by the division of the heat exchanger into a number of small and simple one-pass “mixed-unmixed” crossflow heat exchangers, where the hot fluid was mixed and the cold fluid was unmixed.

**NOMENCLATURE**

- **A** – Heat transfer surface area
- **C** – Heat capacity rate of a fluid
- **c** – Specific heat at constant pressure
- **E** – Heat exchanger effectiveness
- **n** – Number of passes
- **NTU** – Overall number of transfer units
- **r** – Overall heat capacity rate ratio
- **T** – Temperature of the fluid
- **T_A** - Temperature of the external fluid
- **T_{wi}** - Inlet temperature of the tube side fluid
- **T_{wo}** - Outlet temperature of the tube side fluid
- **U_o** - Overall heat transfer coefficient
- **T_w** - Temperature of the tube side fluid
- **T_{Ai}** - Inlet temperature of the external fluid
- **T_{Ao}** - Outlet temperature of the external fluid

**Subscripts**

- **A** – External fluid (hot fluid)
- **o** – Outside
- **O** - Overall
- **w** – Tube side fluid

**Superscripts**

- “” – Quantity expressed on per pass/row basis

2. **THERMAL ANALYSIS**

As mentioned earlier in this document, matrix approach proposed by [1] was directly employed in the thermal analysis of cross flow heat exchangers. Various flow circuiting commonly encountered in industries such as parallel flow, counter flow and neither parallel nor counter flow have been considered in this study. In each instance, a simple heat exchanger consisting of any number of rows and passes has been considered. The number of passes can be visualized as the number of times each tube fluid particle travels through the full extent (length) of heat exchanger and the number of rows per pass can be visualized as the number of split streams in a given pass of a heat exchanger. Both external and tube side fluid may either be treated as mixed or un-mixed in a given pass. The following are the assumptions employed in this study:

1. Fluid properties and thermal properties are constant within the heat exchanger.
2. The heat exchanger is considered to be operating adiabatically.
3. Potential and kinetic energy changes within the heat exchanger are negligible.
4. In case of multiple rows, the fluid streams are split evenly between each row.
5. Effectiveness and NTU are considered to be evenly distributed throughout the heat exchanger.
6. Effectiveness per pass is treated as a constant, though in reality effectiveness per pass could vary due to mass flow rate fluctuations induced by fouling.
In the analysis that follows, the concepts of energy balance and effectiveness shall be applied to every pass of the heat exchanger. The resulting equations are combined in a logical sequence and are subsequently expressed in the form of a matrix. For every flow circuiting, matrices are developed by assuming the tube side fluid as the maximum and also as the minimum capacity rate fluid. Since NTU of the heat exchanger is directly proportional to the heat exchanger surface area, NTU is assumed to be distributed uniformly throughout the heat exchanger. The results from these basic flow circuiting can be considered as building blocks to analyze more complex flow circuiting and assemblies of heat exchangers.

a- PARALLEL CROSS FLOW HEAT EXCHANGER

In this configuration, the principal flow direction of the tube side fluid is same as the flow direction of external fluid. Consider Figure 1, which depicts a two-pass, one row per pass, parallel cross flow heat exchanger.

![Figure 1: Two-pass, one-row per pass parallel cross flow heat exchanger](image)

The tube side fluid temperatures are depicted by subscript w, and hot fluid temperatures are depicted by subscript A. Assuming minimum capacity rate fluid as the tube side fluid, the overall heat capacity rate ratio is defined as

\[ r = \frac{C_{\text{min}}}{C_{\text{max}}} = \left(\frac{m \cdot c}{m \cdot c}\right)_{\text{min}} \]

(1)

The overall capacity rate ratio is assumed to be a known input quantity in this analysis. From Incropera/DeWitt/Bergman/Lavine [17], the overall number of transfer units is defined as

\[ NTU = \frac{U_o \cdot A_o}{C_{\text{min}}} \]

(2)

where \( U_o \) is the overall heat transfer coefficient of the heat exchanger and \( A_o \) is the total outside area of the heat exchanger. In this analysis, NTU is assumed to be a known input quantity.

The overall effectiveness of a heat exchanger varies as a function of \( r \) and NTU, and similarly depends on the mixing conditions assumed within the heat exchanger; standard relations are provided in [17].

However, on a per pass basis, the effectiveness of an individual heat exchanger pass depends on the number of transfer units per pass and the capacity rate ratio per pass, and are expressed as NTU* and \( r^* \) respectively. Considering Figure 1, the capacity rate ratio per pass and the NTU per pass can be defined as

\[ r^* = \left(\frac{m \cdot c}{m \cdot c}\right)_w \]

(3)

\[ NTU^* = \frac{n}{\left(\frac{m \cdot c}{m \cdot c}\right)_A} \]

(4)

where \( n \) is the number of passes (in this case, two).

Rearranging Equations (3) and (4) yields the following equations

\[ r^* = r \]

(5)

\[ NTU^* = \frac{NTU}{n} \]

(6)

Assuming that both the fluids are unmixed in a given pass, the effectiveness per pass can be given as [17]

\[ \varepsilon^* = 1 - \exp\left(\frac{(NTU^*)^{0.22}}{r^*} \left\{ \exp\left( r^*(NTU^*)^{0.78} \right) - 1 \right\} \right) \]

(7)

In the current analysis, the heat exchanger effectiveness is assumed to be as constant for every pass in the heat exchanger. On a global basis, if the tube-side fluid is assumed to be the maximum capacity rate fluid, the overall effectiveness of the heat exchanger can then be defined as [17]

\[ \varepsilon = \frac{T_{A1} - T_{A0}}{T_{A1} - T_{wi}} \]

(8)

where \( T_{A1} \) and \( T_{A0} \) are the inlet and discharge temperatures of the external fluid in the cross flow heat exchanger and \( T_{wi} \) is the inlet temperature of the tube side fluid in the cross flow...
heat exchanger. The above equation can also be extended to determine the effectiveness of a particular pass, wherein, the local fluid inlet and discharge temperatures are employed.

An energy balance on the first pass, expressed in terms of capacity rate ratio per pass, is given by

\[ r^* T_{ai1} + T_{wi1} = r^* T_{ai2} + T_{wi2} \]  \hspace{1cm} (9)

Likewise, performing an energy balance on the second pass yields

\[ r^* T_{ai2} - r^* T_{ao2} + T_{wi2} - T_{wo2} = 0 \]  \hspace{1cm} (10)

Applying an effectiveness relation such as Equation (8) to the first pass yields

\[ T_{ai2} = (1 - e^*_{r1}) T_{ai1} + e^*_{r1} T_{wi1} \]  \hspace{1cm} (11)

Similarly, applying the effectiveness relation to the second pass yields

\[ T_{ai2} = (1 - e^*_{r2}) - T_{ao2} + e^*_{r2} T_{wi2} = 0 \]  \hspace{1cm} (12)

There are four equations and four unknowns in a two pass, one row per pass, parallel cross flow heat exchanger. These four equations can be expressed in matrix form as follows:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
r^* & 0 & 0 & 0 \\
(e^*_{r2} - 1) & 1 & 0 & 0 \\
-r^* & -1 & r^* & 1
\end{bmatrix}
\begin{bmatrix}
T_{ai1} \\
T_{ai2} \\
T_{wi2} \\
T_{wo2}
\end{bmatrix}
= \begin{bmatrix}
(1 - e^*_{r1}) T_{ai1} + e^*_{r1} T_{wi1} \\
0 \\
0 \\
0
\end{bmatrix} \hspace{1cm} (13)

Recognize that \( e^* \) is the same for all the passes and thus may be described as \( E \). Extending the above concept for any number of passes, the matrix for solving the intermediate temperatures and the discharge temperatures for a parallel cross flow heat exchanger may be given as

\[
\begin{bmatrix}
r^* & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
r^* & r^* & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & r^* & -r^* & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1 - E & -1 & 0 & 0 & 0 & E & 0 & 0 & 0 \\
0 & 1 - E & -1 & 0 & 0 & 0 & E & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_{ai1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
T_{ai2} & T_{ai3} & T_{ai4} & T_{ao2} & T_{ao3} & T_{ao4} & T_{twi} & T_{twi} & T_{wo2} \\
T_{wi1} & T_{wi2} & T_{wi3} & T_{wi4} & T_{wi5} & T_{wi6} & T_{wi7} & T_{wi8} & T_{wi9} \\
T_{wo1} & T_{wo2} & T_{wo3} & T_{wo4} & T_{wo5} & T_{wo6} & T_{wo7} & T_{wo8} & T_{wo9}
\end{bmatrix}
= \begin{bmatrix}
(1 - E) T_{ai1} + (ET_{wi1}) \\
(1 - E) T_{ai2} + (ET_{wi2}) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_{ai9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
T_{ao9} & T_{ao10} & T_{ao11} & T_{ao12} & T_{ao13} & T_{ao14} & T_{ao15} & T_{ao16} & T_{ao17} \\
T_{twi10} & T_{twi11} & T_{twi12} & T_{twi13} & T_{twi14} & T_{twi15} & T_{twi16} & T_{twi17} & T_{twi18} \\
T_{wo9} & T_{wo10} & T_{wo11} & T_{wo12} & T_{wo13} & T_{wo14} & T_{wo15} & T_{wo16} & T_{wo17} & T_{wo18}
\end{bmatrix} \hspace{1cm} (14)

These coupled algebraic equations can be solved by using the Gauss elimination method, or any other suitable matrix inversion technique.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & r^* & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & r^* & -r^* & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & r^* & -r^* & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & r^* & -r^* \\
0 & 0 & 0 & 0 & 0 & 1 - E & -1 & 0 & 0 \\
E & 0 & 0 & 0 & 0 & 1 - E & -1 & 0 & 0 \\
0 & E & 0 & 0 & 0 & 0 & 1 - E & -1 & 0 \\
0 & 0 & E & 0 & 0 & 0 & 0 & 1 - E & -1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_{ai1} & T_{ai2} & T_{ai3} & T_{ai4} & T_{ao2} & T_{ao3} & T_{ao4} & T_{ao5} & T_{ao6} \\
T_{ai7} & T_{ai8} & T_{ai9} & T_{ao7} & T_{ao8} & T_{ao9} & T_{ao10} & T_{ao11} & T_{ao12} \\
T_{wi1} & T_{wi2} & T_{wi3} & T_{wi4} & T_{wi5} & T_{wi6} & T_{wi7} & T_{wi8} & T_{wi9} \\
T_{wo1} & T_{wo2} & T_{wo3} & T_{wo4} & T_{wo5} & T_{wo6} & T_{wo7} & T_{wo8} & T_{wo9}
\end{bmatrix}
= \begin{bmatrix}
T_{ai1} + (r^* T_{wi1}) \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_{ai9} & T_{ai10} & T_{ai11} & T_{ai12} & T_{ao2} & T_{ao3} & T_{ao4} & T_{ao5} & T_{ao6} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
T_{wi10} & T_{wi11} & T_{wi12} & T_{wi13} & T_{wi14} & T_{wi15} & T_{wi16} & T_{wi17} & T_{wi18} \\
T_{wo9} & T_{wo10} & T_{wo11} & T_{wo12} & T_{wo13} & T_{wo14} & T_{wo15} & T_{wo16} & T_{wo17} & T_{wo18}
\end{bmatrix} \hspace{1cm} (15)

Similarly, equations can be derived by considering the tube side fluid as the minimum capacity rate fluid. In such a case, the matrix is described as

Equations (14) and (15) can be used to determine the overall and the intermediate thermal performance of any parallel cross flow heat exchanger.
b- COUNTER CROSS FLOW HEAT EXCHANGER

In this configuration, the principal flow of the tube side fluid is opposite to the flow direction of outside fluid. Consider Figure 2, which depicts a two-pass, one row per pass counter cross flow heat exchanger.

Tube side fluid is denoted by the subscript ‘w’ and the external fluid is represented by the subscript ‘A’.

First, an analysis is performed by considering the tube side fluid as the maximum capacity rate fluid. An energy balance on the first pass, expressed in terms of capacity rate ratio per pass, is given by

\[ r^*T_{A12} - r^*T_{w12} + T_{wo} = r^*T_{Ai} \]  \hspace{1cm} (16)

Likewise, performing an energy balance on the second pass yields

\[ r^*T_{A12} - r^*T_{A0} - T_{w12} + T_{wi} = 0 \]  \hspace{1cm} (17)

Applying an effectiveness relation such as Equation (8) to the first pass yields

\[ T_{A12} - \varepsilon^*_{r1}T_{w12} = T_{Ai}(1-\varepsilon^*_{r1}) \]  \hspace{1cm} (18)

Similarly, applying the effectiveness relation to the second pass yields

\[ T_{A12}(1-\varepsilon^*_{r2}) - T_{A0} + \varepsilon^*_{r2}T_{wi} = 0 \]  \hspace{1cm} (19)

There are four equations and four unknowns in a two pass, one row per pass counter cross flow heat exchanger. These four equations can be expressed in a matrix form as follows:

\[
\begin{bmatrix}
1 & 0 & -\varepsilon^*_{r1} & 0 \\
(e^*_{r2} - 1) & 1 & 0 & 0 \\
-r^* & r^* & 1 & 0 \\
r^* & 0 & -1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
T_{A12} \\
T_{A0} \\
T_{w12} \\
T_{wo} \\
\end{bmatrix}
= \begin{bmatrix}
(1-\varepsilon^*_{r1})T_{Ai} \\
T_{wi} \\
r^*T_{Ai} \\
\end{bmatrix}
\]  \hspace{1cm} (20)

Recognize that \( \varepsilon^* \) is the same for all the passes and thus may be described as E. Extending the above concept for any number of passes, the matrix for solving the intermediate temperatures and the discharge temperatures for a counter cross flow heat exchanger may be given as

\[
\begin{bmatrix}
r^* & 0 & 0 & 0 & \cdots & -1 & 0 & 0 & 0 \\
r^* & -r^* & 0 & 0 & \cdots & -1 & 1 & 0 & \cdots \\
0 & r^* & -r^* & 0 & \cdots & -1 & 1 & 0 & \cdots \\
0 & 0 & r^* & -r^* & \cdots & 0 & 0 & -1 & \cdots \\
0 & 0 & 0 & r^* & -r^* & \cdots & 0 & 0 & 0 & \cdots \\
1 & 0 & 0 & 0 & \cdots & -E & 0 & 0 & \cdots \\
1 - E & 1 & 0 & 0 & \cdots & 0 & -E & 0 & \cdots \\
0 & 1 - E & 1 & 0 & \cdots & 0 & 0 & -E & \cdots \\
0 & 0 & 0 & 1 - E & 1 & \cdots & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
T_{A12} \\
T_{A0} \\
T_{w12} \\
T_{wo} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
= \begin{bmatrix}
(1-\varepsilon^*_{r1})E_{T_{Ai}} \\
E_{T_{wi}} \\
E_{T_{wi}} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix}
\]  \hspace{1cm} (21)

Similarly, equations can be derived by considering the tube side fluid as the minimum capacity rate fluid. In such a case, the matrix is described as follows
Equations (21) and (22) can be used to determine the overall and the intermediate thermal performance of any counter cross flow heat exchanger.

**c- NEITHER PARALLEL NOR COUNTER CROSS FLOW HEAT EXCHANGER**

In this flow configuration, the flow can neither be classified as counter cross flow nor parallel cross flow. Consider Figure 3 depicting a one pass two rows per pass neither parallel nor counter cross flow heat exchanger.

Herein, the tube side fluid is assumed to be equally divided between the two rows. In the analysis that follows, the tube side fluid is considered as the maximum capacity rate fluid.

![Figure 3](https://example.com/figure3.png)

The discharge temperatures for a neither parallel nor counter cross flow heat exchanger may be given as:

\[ T_{w1} = T_{w2} \]

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-r^* & 0 & 0 & 0 \\
r^* & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_{Ai} \\
T_{A12} \\
T_{A23} \\
T_{A34} \\
\end{bmatrix}
= \begin{bmatrix}
r^*T_{Ai} + T_{wi} \\
r^*T_{A12} + T_{wi} \\
r^*T_{A23} + T_{wi} \\
r^*T_{A34} + T_{wi} \\
\end{bmatrix}
\]

Recognize that \( r^* \) is the same for all the rows and thus may be described as \( E \). Extending the above concept for any number of rows, the matrix for solving the intermediate temperatures and the discharge temperatures for a neither parallel nor counter cross flow heat exchanger may be given as:

\[
\begin{bmatrix}
r^* & 0 & 1 & 0 & 0 & 0 \\
-r^* & 0 & 0 & 0 & 0 & 0 \\
r^* & 0 & -1 & 0 & 0 & 0 \\
r^* & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_{Ai} \\
T_{A12} \\
T_{A23} \\
T_{A34} \\
T_{A45} \\
\end{bmatrix}
= \begin{bmatrix}
r^*T_{Ai} + T_{wi} \\
r^*T_{A12} + T_{wi} \\
r^*T_{A23} + T_{wi} \\
r^*T_{A34} + T_{wi} \\
r^*T_{A45} + T_{wi} \\
\end{bmatrix}
\]

Likewise, performing an energy balance on the second row yields

\[
r^*T_{A12} - r^*T_{A23} - T_{w2} = -T_{wi}
\]

Applying an effectiveness relation such as Equation (8) to the first row yields

\[
(\varepsilon^*_r - 1)T_{Ai} - \varepsilon^*_r T_{wi} = 0
\]

Similarly, applying the effectiveness relation to the second row yields

\[
(\varepsilon^*_r - 1)T_{A12} + T_{A23} = \varepsilon^*_r T_{wi}
\]

These four equations can be expressed in a matrix form as follows:
Similarly, equations can be derived by considering the tube side fluid as the minimum capacity rate fluid. In such a case, the matrix is described as follows:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & r^* & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & -r^* & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & -r^* & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & -r^* & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -r^* & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
E -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & E -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & E -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{TA}12 & \text{TA}23 & \text{TA}34 & \text{TA}45 & \text{TA} & \text{ET}i \\
\text{rT}Ai + \text{Twi} & -\text{Twi} & -\text{Twi} & -\text{Twi} & -\text{Twi} & -\text{Twi} \\
\text{Twi} & \text{Twi} & \text{Twi} & \text{Twi} & \text{Twi} & \text{Twi} \\
\text{E} - \text{1} \times \text{T}Ai + \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i \\
\text{E} - 1 & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i \\
\text{E} - 1 & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i \\
\text{E} - 1 & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i & \text{ET}i \\
\end{bmatrix}
\]

(28)

Equations (28) and (29) can be used to determine the overall and the intermediate thermal performance of any neither parallel nor counter cross flow heat exchanger.

### d- PHASE CHANGE

Assume a cross flow heat exchanger wherein the tube side fluid undergoes a phase change from saturated vapor to saturated liquid while the external fluid undergoes only sensible heat transfer. In such an instance, the capacity rate ratio shall be zero, and the fluid undergoing sensible heat transfer is the minimum capacity rate fluid. NTU is a known input quantity and is assumed to be uniformly distributed throughout the heat exchanger. Thus, the NTU per row/pass can be readily determined. Using standard relations and charts [17], the effectiveness per row/pass can be determined as well.

The concept of energy balance and effectiveness for each pass/row can now be employed to yield the intermediate temperatures for the external fluid and the intermediate enthalpies of the tube side fluid. For any given heat exchanger geometry, such an exercise can be readily modeled and this shall help the designers to determine the number of rows/passes required for a phase change to occur in the tube side fluid.

### CONCLUSIONS

Generalized equations have been developed to study the steady state performance of a cross flow heat exchangers. Geometries commonly seen in industries such as parallel cross flow, counter cross flow and neither parallel nor counter cross flow have been addressed in this research work. Both sensible heat transfer and phase change have been considered in this research work. Generalized equations have been developed by considering the tube side fluid as both the maximum and minimum capacity rate fluid. The generalized equations have been expressed in a logical matrix format. By solving the matrix equations, the thermal performance of the heat exchanger at the intermediate level and at the global level can readily be determined. Since the research work uses the concept of NTU, this work can be universally applied to any cross flow heat exchanger. Also, the intermediate and global thermal behavior of the heat exchanger can be readily determined for various materials, thermal and flow characteristics, various heat exchanger designs and sizes. Therefore this research work will help in building the most cost efficient and an optimum heat exchanger for a given application. Availability of intermediate thermal details will additionally help in understanding thermal stresses in the heat exchanger during the conceptual phase. This work is very applicable during the design (prior to prototype building) phase of the heat exchanger. In-addition, during the
operational phase, an underperforming heat exchanger can immediately be spotted and the equipment can be scheduled for required maintenance work. It should be noted that the work presented must be considered as building blocks to determine the thermal performance of more complex heat exchangers.

REFERENCES


