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Abstract—This paper presents theoretical formula, based on the theory of multi-phase mixture, for the prediction of critical slope angle for which the sediment deposits rested on an infinite slope will be initiated to be debris flows. The deposit sediment is assumed to be a mixture of four constituents, i.e., stone, mud, water and air. Two-dimensional steady uniform flow field is considered. Critical slope angles can be calculated from the consideration of momentum equations or static balance of driving force and resisting force. Comparison of the present results with those obtained by seepage flow theory, Takahashi theory and modified Takahashi theory shows that the theoretical formula derived from multi-phase mixture theory gives the same prediction as Takahashi theory and can be extended to the sediments containing more than two kinds of constituents. Variation of critical angle slope with volume fraction of gravel and sand for stony flow, mud flow and mixed flow is also conducted and the results are reasonable as expected.

Keywords—Critical Slope Angle, Debris Flow, Flow Initiation, Multi-phase Mixture Theory, Theoretical Analysis.

I. INTRODUCTION

Landslide and debris flow often occur in the region where deposit and mud saturated after strong and/or continuous stormy rainfall. The initiation of mudslide can be considered to be a mechanism of unbalance of gravitational force (driving force) and the shearing force (frictional force). The critical slope angle is defined to be the smallest angle that the gravitational force of deposit greater than the frictional force and thus unstable condition is reached and deposit starts to move [1, 2]. The analysis of the inherent characteristics of the debris flows and the development of prevention and precaution techniques are urgently required. Correct analysis of dynamic characteristics of debris flow is a fundamental basis for designing effective Sabo dams for prevention of debris flows.

Occurrence of debris flow depends highly on local topographic, meteorologic, geologic, and hydrologic conditions. However, there are many factors influence the occurrence of debris flow in a region. For example, the special features and natural reasons that cause debris flows often occur in Taiwan:

1) Climate factors: Taiwan is located at the border of western Pacific and the eastern Asian plateau and belongs to subtropical marine climate. Yearly averaged rainfall reaches a high value of 2500 mm. Many typhoons occur in Taiwan between July and October (concentrated from summer to autumn) each year and carry with tremendous rainfall and strong wind and then lead to flood in rivers and softening of the slope of mountain. The storms cased from typhoons, thunder shower, thunder cell, and stationary front usually leads to continuously tremendous rainfalls which in turn soften the soil stratum and rock in mountain area. This is the first influence factor leads to highly potential of debris flow disaster.

2) Tectonics factors: Taiwan is located at the intersection of Eurasian Plate and Philippine Sea Plate. The upper part of the crust is primarily made from a series of collision of these two plates. The Philippines Sea Plate is converging with the continent at 7 cm per year in the western north direction. This cause many seismic faults, fracture surfaces, folded zones, mud volcanoes and hot springs in Taiwan. The mountain area plays an important part of Taiwan up to 75% and often contains steepest slope angles, high crest and quick flow along with long-term strong erosion. This forms the first influence factor leads to highly potential of debris flow disaster.

3) Seismic factors: Taiwan is also located at the Circum-Pacific Seismic Belt and thus many earthquakes occur each year. The horizontal and vertical accelerations induced by earthquakes usually leads to liquefaction of perfectly and/or partially saturated stratum as well as softening, peel-off, separation and deposit of rocks. When waterfall combined with these deposits might leads to the origin of debris flow.

From the summarized factors we can observe a common feature among these three factors that is the existence of tremendous mountain area which contains steepest slope angle and thus enter the range of critical angle of occurrence of debris flow, from 15° to 25°.
There have been a lot of flow models for the analysis of mechanical characteristics of debris flow in a slope. Among these research works, the following models are useful and significant: the dilatant fluid model initiated by Bagold (1954) [3] and extended by Takahashi (1977, 1978) [4, 5]; the Bingham fluid model, pseudo- or generalized viscoplastic fluid model such as those proposed by Chen (1986) [6], O’Brien and Julien (1988) [7], Chen et al. (1991) [8], Julien and Lan (1991) [9], etc.; Prandtl mixing-length model employed by Matsumura and Mizuyama (1990) [10]; modified turbulent flow model proposed by Yu and Chen (1990) [11]; a mixed-layer model proposed by Su et al. (1993) [12]; a two-layer model, proposed by Ho (1997) [13] in which an inertia sub-region and a viscous sub-region exist.

In the design of disaster prevention of debris flows the accurate prediction of initiation of debris flows from unsaturated or saturated deposits along a slope is of the most importance. From this the analysis of movement, stoppage, deposit and impact on structures can be studied. There also are many theoretical formula for the prediction of the critical slope for the initiation of debris flow such as the seepage flow theory by Harris (1977) [14], Empty pore pressure theory by Takahashi (1977) [4], modified seepage flow theory by Sidle (1985) [2], Zheng and Chiang (1986) [15]; modified Takahashi theory by Yu (1987) [16], extended seepage flow theory by Huang (1991) [17], incompressible uniform flow model by Egashira and Ashida (1992) [18], Combined run-off and seepage flow theory by Lin et al. (1993) [19], and varied slope and thickness theory by Chen et al. (1993) [20]. The fundamental defects existing in the present formula arise from the assumption of the constituents of debris flows, i.e. only solid and fluid particles are considered. In fact most debris flows comprise more than two kinds of composites such as stone (larger particles), mud (smaller particles), water and air. The constitutive (rheological) relationships for each of them are quite different.

Thus we can develop a rheological model based on the theory of multi-phase mixture, in which the debris flow is considered to be a mixture of four constituents, i.e., stone, mud, water and air. The total shear stress is the sum of the shear stresses contributed from each phase through its volume fraction. Once the constitutive relationship is built up we can theoretically derive the critical slope angle for debris flow along an infinite slope based on momentum equations or static balance of driving force and resisting force.

II. THREE DISASTER CASES OF DEBRIS FLOW IN TAIWAN

The recorded investigation on these three real disasters caused by debris flow is summarized in Table I. It is very important that the slope angles in the source area (initiation zone) of these three disasters lie between 15° to 25°. This range is the easiest occurrence of debris flow.

<table>
<thead>
<tr>
<th>Period</th>
<th>Place</th>
<th>Type of debris flow</th>
<th>Cause</th>
<th>Slope angles</th>
<th>Source Area</th>
<th>Flow Track</th>
<th>Deposition Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996/07/31~08/01</td>
<td>Nan-Tou, Shin-Yi, Shen-Mu Village</td>
<td>Mixed</td>
<td>Typhoon</td>
<td>18°~21°</td>
<td>10°~20°</td>
<td>6°~9°</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>Deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>56,000 m³</td>
</tr>
<tr>
<td>IV</td>
<td>122,000 m³</td>
</tr>
<tr>
<td>IV</td>
<td>450,000 m³</td>
</tr>
</tbody>
</table>

III. CRITICAL SLOPE ANGLE PROPOSED BY PREVIOUS RESEARCHERS

In the following we summarized the formula for critical slope angle of occurrence of debris flow proposed by previous researchers:

\[
\tan \theta \geq \frac{\gamma_{sat} - \gamma_w \cdot \tan \phi_S}{\gamma_{sat}}
\]

where

\[
\begin{align*}
\theta & : \text{slope angle of open channel} \\
\phi_S & : \text{angle of internal friction of deposit} \\
\gamma_{sat} & : \text{saturated unit weight of deposit} \\
\gamma_w & : \text{unit weight of water}
\end{align*}
\]
(2) Empty pore pressure theory by Takahashi (1977) [4]:
\[
\tan \theta \geq \frac{(1-n)(\gamma_d - \gamma_w)}{(1-n)(\gamma_d - \gamma_w) + \gamma_w(1+H)/h} \cdot \tan \phi_S
\]  
(2)
where
\[
\theta : \text{slope angle of open channel}
\]
\[
\phi_S : \text{angle of internal friction of deposit}
\]
\[
\gamma_d : \text{unit weight of deposit}
\]
\[
\gamma_w : \text{unit weight of water}
\]
\[
H : \text{depth of the water}
\]
\[
h : \text{depth of layer of shear failure}
\]
(3) Modified seepage flow theory by Sidle (1985) [2]:
\[
\tan \theta \geq \frac{\gamma_w \gamma_m a + (\gamma_w - \gamma_m)(H-a)\tan \phi_S}{\gamma_w \gamma_m a + \gamma_w (H-a)}
\]  
(3)
(4) Modified seepage flow theory by Zheng and Chiang (1986)[15]:
\[
\tan \theta \geq \frac{c l q H \cos \theta + C_d [\gamma_d - \gamma_w(1-a/H)]}{C_d [\gamma_d - \gamma_w(1-a/H)] + \gamma_w(1-a/H)}
\]  
(4a)
\[
\tan \theta \geq \frac{c l q H \cos \theta + C_d [\gamma_d - \gamma_w]}{C_d [\gamma_d - \gamma_w] + \gamma_w(a + h_w/H)}
\]  
(4b)
(5) Modified Takahashi theory by Yu (1987) [16]:
\[
\tan \theta \geq \frac{1}{1 + \frac{\gamma_w(H-a)}{\gamma_sat H - n \gamma_w(H-a)}} \tan \phi
\]  
(5)
(6) Extended seepage flow theory by Huang (1991, 1993) [17, 18]:
\[
\tan \theta \geq \frac{\gamma_w - \gamma}{\gamma_sat} \cdot \tan \phi_S
\]  
(6a)
\[
\tan \theta \geq \frac{\gamma_w - \gamma}{\gamma_sat} \cdot \tan \phi_S
\]  
(6a)
(b) run-off under surface of debris flow:
\[
\tan \theta \geq \frac{\gamma_w a + \gamma_sat(H-a)}{\gamma_w a + \gamma_sat(H-a) + \gamma_w H} \cdot \tan \phi_S
\]  
(6b)
(7) Incompressible uniform flow model by Egashira and Ashida (1992) [19]:
\[
\tan \theta = \frac{1}{2(\rho_s / \rho_w - 1)C[\tan \phi_S/(1+\alpha) - \tan \theta]}
\]  
(7)
(8) Combined run-off and seepage flow theory by Lin et al. (1993) [20]:
\[
\tan \theta \geq \frac{(\gamma_sat - \gamma_w)H}{\gamma_sat H + \gamma_w h_0} \tan \phi_S
\]  
(8)
(9) Varied slope and thickness theory by Chen et al. (1993) [21].
\[
\begin{align*}
\text{(a) Sliding Instability case:} & \quad \text{tanh} \left( \frac{C(\gamma_d - \gamma_w) \tan \theta}{C(\gamma_d - \gamma_w) + \gamma_w(a + h_w) + \gamma_w H} \right) \\
& \quad + \frac{d h_0}{d x} + \frac{d h_w}{d x} + \frac{\tau_0}{\rho \cos \theta}
\end{align*}
\]  
(9a)
\[
\begin{align*}
\text{(b) Overturning Instability case:} & \quad \text{tanh} \left( \frac{C(\gamma_d - \gamma_w) \tan \theta}{C(\gamma_d - \gamma_w) + \gamma_w(a + h_w) + \gamma_w H / 3} \right) \\
& \quad + \frac{1}{2} \frac{d h_0}{d x} + \frac{d h_w}{d x}
\end{align*}
\]  
(9b)

It should be noticed that these formula are derived from different consideration and may not be compared with one another.

IV. MODEL EQUATIONS FOR DEBRIS FLOWS

Consider a mass of sediment deposits at an infinite slope with angle \( \theta \). In the beginning the sediment is at rest. When the sediment is gradually saturated and the slope angle is large enough to cause the sediment starts to move down the plane, the slope angle is termed critical boundary for stability. The depth of deposit is \( H \) with the coordinate system selected as shown in Figure 1.
4.1 Basic assumptions

In the formulation of the governing equations of debris flow some assumptions are employed as depicted in Huang and Hsiao [22]. Furthermore, cohesion of muddy sediments is neglected when critical condition is considered.

4.2 Macroscopic balance equations for debris flow

The debris flow is composed of stone, mud, water and air. Thus the density of the mixture can be expressed as

\[ \rho = \eta_g \rho_g + \eta_m \rho_m + \eta_w \rho_w + \eta_a \rho_a \]

\[ = \sum_{\alpha=1}^{4} \eta_{\alpha} \rho_{\alpha} \]  

(10)

Where \( \rho_g, \rho_m, \rho_w, \rho_a \) denotes the density of stone, mud, water and air, respectively. It is obviously seen that \( \sum_{i=1}^{4} \eta_{\alpha} = 1 \).

Follow the derivation of balance equations of the mixture as proposed by Hassanizadeh and Gray (1979, 1980) [22], we can obtain the following balance equations in indicial form:

\[ \frac{D^{2}}{D t^{2}}(\eta_{\alpha} \rho_{\alpha}) + \eta_{\alpha} \rho_{\alpha} v_{k}^{\alpha} = 0 \]  

(11a)

\[ \rho \frac{D}{D t} v_{l}^{\alpha} - \alpha_{k,l}^{\alpha} - \eta_{\alpha} \rho_{\alpha} Q_{l}^{\alpha} - \eta_{\alpha} T_{l}^{\alpha} = 0 \]

(11b)

\[ \alpha_{k,l} = \alpha_{l,k} \]

(11c)

\[ \eta_{\alpha} \rho_{\alpha} \frac{D^{2} E_{\alpha}}{D t^{2}} - \alpha_{k,l}^{\alpha} \alpha_{l}^{\alpha} = 0 \]  

(11d)

\[ \eta_{\alpha} \rho_{\alpha} \frac{D S_{\alpha}}{D t} - \alpha_{k,l}^{\alpha} \alpha_{l}^{\alpha} = \eta_{\alpha} \rho_{\alpha} \Gamma_{\alpha} \]  

(11e)

subjected to

\[ \sum_{\alpha=1}^{4} \eta_{\alpha} \rho_{\alpha} v_{l}^{\alpha} = 0 \]  

(12a)

\[ \sum_{\alpha=1}^{4} \eta_{\alpha} \rho_{\alpha} (v_{l}^{\alpha} + Q_{l}^{\alpha}) = 0 \]  

(12b)

When the temperature in each phase is the same (isothermal condition), i.e.,

\[ \theta_g = \theta_m = \theta_w = \theta_a = \theta \]  

(13)

As a result, energy equations for each phase need not be considered. Rather a total energy equation obtained as the sum of each phase energy equations will be

\[ \rho \frac{D E}{D t} = \sum_{\alpha=1}^{4} \alpha_{k,l}^{\alpha} + \eta_{\alpha} \rho_{\alpha} E_{\alpha} = \sum_{\alpha=1}^{4} \alpha_{k,l}^{\alpha} \]

(14)

and

\[ \rho h = \sum_{\alpha=1}^{4} \rho_{\alpha} h_{\alpha} \]  

(16)

The definition of all variables in the above balance equations of a multi-phase system can be found in [22]. Since the problem is two-dimensional and steady uniform flow is assumed thus in the above governing equations \( D^{2}() / D t^{2} = v_{k} \bullet \text{grad}() \), i.e., \( \partial \) / \( \partial t = 0 \) and the indices \( k,l \) run from 1 to 2. Furthermore, \( E,h,\theta \) etc., are constants in this situation and only mass and momentum equations (in \( x, y \) directions) should be taken into account.
For a mixture of immiscible solids and fluids it is reasonable to use the volume fraction in the whole volume to replace the volume fraction of a representative element volume, i.e.,

$$C_\alpha = \frac{V_\alpha}{V} = \frac{dV_\alpha}{dV} \quad (\alpha = g, m, w, a) \quad (17)$$

### 4.3 Constitutive relationship

The shear stress in the debris flow can be considered as the superposition of those resulting from the solid phases (stone and mud) and the fluid phases (water and air).

$$\tau = \tau_g + \tau_f$$

$$= (C_g \tau_g + C_m \tau_m) + (C_w \tau_w + C_a \tau_a)$$

in which $\tau_g$ is the contribution of shear stress from stony phase and, based on the dilatant fluid model conducted by Bagnold (1954) and Takahashi (1977), can be expressed as [3, 4]

$$\tau_g = \frac{\tau_g^y}{g} + \frac{\eta_g}{g} \left( \frac{d\tau_g}{dy} \right)^2 = -P_s \tan \phi_{gs} + \frac{\eta_g}{g} \left( \frac{d\tau_g}{dy} \right) \quad (19a)$$

Where $\tau_g^y = -P_s \tan \phi_{gs}$ denotes the yield stress of grain with $P_s$ being the normal stress and $\phi_{gs}$ the static friction of sediment; $\eta_g = a \sin \phi_0 \rho_s d_s^2 \lambda^2$ with $a$ being a constant, $\phi$ the kinetic friction of sediment, $\rho_s$ the density of sediment, $d_s$ the diameter of particle of sediment, $\lambda$ the characteristic length related to the concentration. It is noticed that $\eta_g$ is a factor represents the contribution of dispersive pressure of particles in which grain inertia effect dominates [3-5].

$\tau_m$ is the contribution of shear stress from mud phase and, based on the Bingham fluid model conducted by Takahashi (1977), can be expressed as [4, 5]

$$\tau_m = \frac{\tau_m^y}{m} + \frac{\eta_m}{m} \left( \frac{d\tau_m}{dy} \right) = -P_s \tan \phi_{ms} + c' \frac{d\tau_m}{dy} + \eta_m \left( \frac{d\tau_m}{dy} \right) \quad (19b)$$

Where $\tau_m^y = -P_s \tan \phi_{ms}$ denotes the yield stress of mud with $P_s$ being the normal stress and $\phi_{ms}$ the static friction of mud. It is well known that the cohesion of the mud, $c'_{ms}$, is usually very small and can be neglected. It is noticed that this is a linear relationship between the shear stress and rate of deformation.

It is noticed that $\tau_w, \tau_a$ is the contribution of shear stress from water phase and air phase, respectively; which can be considered to be Newtonian fluids and thus [1, 2]

$$\tau_w = \mu_w \left( \frac{d\tau_w}{dy} \right) \quad (19c)$$

$$\tau_a = \mu_a \left( \frac{d\tau_a}{dy} \right) \quad (19d)$$

Where $\mu_w, \mu_a$ are the dynamic viscosity of water and air, respectively. From Eq. (19c) and (19d) we can find that there is no yield stresses in the Newtonian fluids (water and air).

### V. CRITICAL CONDITION OF SLOPE ANGLE

#### 5.1 Derivation from the balance equations:

For a two-dimensional steady uniform debris flow the continuity equation implies that the velocity field is $u = u(y), v \equiv 0$ in $x$ and $y$ direction, respectively. The momentum equations can be rewritten in explicit form as

$$x: \quad \rho \frac{Du}{Dt} = 0 = \rho g \sin \theta + \frac{\partial \tau_{XY}}{\partial \tau_{XY}} \quad (20a)$$

$$y: \quad \rho \frac{Dv}{Dt} = 0 = -\rho g \cos \theta - \frac{\partial p}{\partial y} \quad (20b)$$

It can be deduced that the pressure is $p(y) = \rho f g \cos \theta (H - y)$.

Integrating Eqs. (20a) and (20b) from $y$ to $H$ and from the steady versions of the constitutive relationships (19), we can derive that the condition for the sediment to start to move:
\[
\tan \theta = \frac{C_g \rho_g + C_m \rho_m - (1 - C_w) \rho_w - (1 - C_a) \rho_a}{C_g \rho_g + C_m \rho_m + C_w \rho_w + C_a \rho_a} \cdot \tan \phi_s
\]

(21)

5.2 Derivation from the static balance:
Considering the typical sediment along an infinite slope with height \( H \), length \( b \) and unit thickness (Fig. 1). The driving force due to gravitational force is

\[
T = \rho_g \sin \theta (b)(H)(1)
= (C_g \rho_g + C_m \rho_m + C_w \rho_w + C_a \rho_a) b H g \sin \theta
\]

(22)

The resisting force due to granular and mud sediment can be expresses as

\[
R = (\rho - \rho_f) g \cos \theta \tan \phi_s (b)(H)(1)
= [C_g \rho_g + C_m \rho_m - (1 - C_w) \rho_w - (1 - C_a) \rho_a] \cdot b H g \cos \theta \tan \phi_s
\]

(23)

Sediment begins to move when the driving force exceeds the resisting force, i.e., \( T \geq R \). Thus we have the critical condition the same as Eq. (21).

VI. RESULTS AND DISCUSSION

In order to check the validity of theoretical expression of critical slope angle obtained by the theory of multi-phase mixture, two groups of cases are considered as testing examples each of which contains three different sediment constituents:

6.1 Sampled data

The sampled specimen are referred to Yu (1987) and listed in Table II. Three sediments include natural deposits, uniformly distributed gravels and coarse sands, respectively. The critical slope angles evaluated from the seepage flow theory (Harris 1977) [14], Takahashi theory (Takahashi 1977) [4], modified Takahashi theory (Yu, 1987) [16], and the present formula based on multi-phase mixture theory, Eq. (21), are also summarized in Table 1.

It is noticed that for three samples the present theory gives the same prediction as the Takahashi theory, while seepage flow theory yields non-conservative results but the modified seepage flow theory gives too conservative critical slope angles.

6.2 Three kinds of sediments initiated to different debris flows

In Table III the critical slope angles for three different types of sediment constituents deposits in infinite slope are analyzed based on seepage flow theory, Takahashi theory, modified Takahashi flow theory, and the present theory, Eq. (21). We found that only modified Takahashi flow theory gives different and conservative predictions while the other three agree well with one another. The very low critical slope occurs in mud flow due to its low static friction angle of sands.

To examine the variation of critical slope angle with the volume fractions of three different kinds of debris flow, we change the parameters and calculate the critical slope angles from Eq. (21).

Figure 2 shows that variation of critical slope angle with the volume fraction of gravel, \( C_g \), for stony flow. It is obvious that the relationship is nonlinear and the larger the volume fraction is the larger the critical slope angle. This means the less content of water, the more stable of the sediment.

Figure 3 shows that variation of critical slope angle with the volume fraction of sand, \( C_m \), for mud flow. It is obvious that the relationship is nonlinear too and the larger the volume fraction is the larger the critical slope angle. This means the less content of water, the more stable of the sediment.

Figure 4 shows that variation of critical slope angle with the volume fraction of gravel, \( C_g \), for mixed flow in which the volume fraction of sand is kept constant. It is obvious that the relationship is nonlinear and the larger the volume fraction is the larger the critical slope angle. This means the less content of water, the more stable of the sediment for mixed flow too.
### TABLE II
THREE SAMPLED DEPOSITS AND THEIR CORRESPONDING CRITICAL SLOPE ANGLE BASED ON VARIOUS THEORIES

<table>
<thead>
<tr>
<th>Sample</th>
<th>Soil</th>
<th>$d_{50}$ (mm)</th>
<th>$\rho_s$ (kg/m$^3$)</th>
<th>$\rho_{sat}$ (kg/m$^3$)</th>
<th>$\rho_m$ (kg/m$^3$)</th>
<th>$n$</th>
<th>$\phi_s$</th>
<th>$\theta_{crit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Natural deposits</td>
<td>2.5</td>
<td>2700</td>
<td>2020</td>
<td>1630</td>
<td>0.39</td>
<td>35°</td>
<td>19.47°</td>
</tr>
<tr>
<td>B</td>
<td>Uniform gravels</td>
<td>7.0</td>
<td>2500</td>
<td>1730</td>
<td>1430</td>
<td>0.42</td>
<td>32°</td>
<td>14.77°</td>
</tr>
<tr>
<td>C</td>
<td>Coarse sands</td>
<td>0.62</td>
<td>2660</td>
<td>1850</td>
<td>1580</td>
<td>0.39</td>
<td>37°</td>
<td>16.02°</td>
</tr>
</tbody>
</table>

**Table Legend:**
- Seepage flow theory (Harris, 1977), Eq. (1)
- Takahashi Theory (Takahashi, 1977), Eq. (2)
- Modified Takahashi theory (Yu, 1987), Eq. (5)
- Multi-phase Mixture theory (Present) Eq. (21)

### TABLE III
THREE DIFFERENT DEBRIS FLOWS AND THEIR CORRESPONDING CRITICAL SLOPE ANGLE BASED ON VARIOUS THEORIES

<table>
<thead>
<tr>
<th>Case</th>
<th>Sediment type</th>
<th>$\rho_s$ (kg/m$^3$)</th>
<th>Volume Fractions</th>
<th>$\phi_s$</th>
<th>$\theta_{crit}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Stony</td>
<td>2650</td>
<td>$C_g$ 0.45, $C_m$ 0.0, $C_w$ 0.55</td>
<td>32°</td>
<td>14.90°, 14.91°</td>
</tr>
<tr>
<td>B</td>
<td>Muddy</td>
<td>2650</td>
<td>$C_g$ 0.0, $C_m$ 0.39, $C_w$ 0.61</td>
<td>5°</td>
<td>1.96°, 2.55°</td>
</tr>
<tr>
<td>C</td>
<td>Mixed</td>
<td>2650</td>
<td>$C_g$ 0.26, $C_m$ 0.50, $C_w$ 0.24</td>
<td>38°</td>
<td>23.49°, 27.57°</td>
</tr>
</tbody>
</table>

**Table Legend:**
- Seepage flow theory (Harris, 1977), Eq. (1)
- Takahashi Theory (Takahashi, 1977), Eq. (2)
- Modified Takahashi theory (Yu, 1987), Eq. (5)
- Multi-phase Mixture theory (Present) Eq. (21)

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**Figure 2.** Variation of critical slope angle with volume fraction of gravel, $C_g$, for stony flow

**Figure 3.** Variation of critical slope angle with volume fraction of sand, $C_m$, for mud flow
VII. CONCLUDING REMARKS

Theoretical formula for evaluating the critical slope angle for initiation of a two-dimensional steady uniform debris flow along an infinite slope is derived based on the theory of multi-phase mixture. The debris flow is in general assumed to be a mixture of four constituents, i.e., stone, mud, water and air. Critical slope angles can be calculated from the consideration of momentum equations or static balance of driving force and resisting force. Comparison of the present results with those obtained by seepage flow theory, Takahashi theory and modified Takahashi theory shows that the theoretical formula derived from multi-phase mixture theory gives the same prediction as Takahashi theory and can be extended to the sediments containing more than two kinds of constituents.

REFERENCES