Development of Gearbox for High Speed Vertical Centrifugal Pump

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Abstract—Vibration and noise analysis as well as strength of gear teeth, roller bearing life, journal bearing design are considered in order to the high speed vertical centrifugal pump which had a speed increaser. Also strict API standard were introduced for reliability evaluation of the developed gearbox, and performance evaluation were carried out. The result that evaluation items about bearing vibration, shaft vibration, noise, and lubrication temperature were selected, and were tested, a high speed vertical centrifugal pump were able to correspond to all API standard.

Keywords—gearbox, vertical centrifugal pump, performance, noise, vibration, lubrication temperature

I. INTRODUCTION

The driveline of high speed rotating machinery uses the directly connected motor or the speed increaser gear. Most of recent machines running during successively long period have the speed increaser which is more reliable.

Gearbox can be grouped into two classes; the speed increaser and the speed reducer. Since the application of the speed increaser has relatively narrow usage unlike the speed reducer, the technical database of designing the speed increaser is insufficient.

There are wide researches on designing gearbox: Lee designed the speed reducer improved the noise and vibration by optimizing theorem.[1] Lida investigated that the dynamic behavior of the gear-shaft system considering bending and torsional coupled effect in spur gears is different from the result considering the system as a non-coupled simple model.[2] Schwibinger found that this coupling of bending and torsion in spur gears affects the stability of the gear-shaft system. [3] Choy developed a dynamic model of the 3-speed spur gear-rotor system forced by mass unbalance with consideration of the bending and torsional coupled effect, then calculated the solutions of transient and steady state responses. [4] And Choy modeled the reduction gear box with the 1-speed spur gear pair by the transfer matrix method at the parts of gear-rotor-bearing and the FEM at the case, then compared the calculated results of vibrational spectra with that of an experiment.[5]

Kahraman solved the critical speed considering coupled effect between torsional and bending vibration of the single stage spur gear, and used FEM to calculate the response of mass unbalance and gear transmission error.[6] Lee predicted the torsional vibration and noise source of 8-stage gas turbine for generator using transfer matrix of Hibner’s branch method.[7]

In this work, we analyzed gear strength based on AGMA standard[8], designed gearbox for high speed vertical centrifugal pump with noise and vibration consideration, and manufactured the prototype. We also evaluated the performance and reliability of the prototype based on the API standard strictly.[9] The experimental result showed the prototype satisfied all the API standard aspects; vibration of bearing and shaft, noise and lubrication temperature.

II. LAYOUT

A. Bending Strength

The bending strength of the tooth root fillet against the loading force transmitted through gear chain is decided by several factors. Those are the tangential component of the loading force on tooth surface, the compressive stress by the component of radial force, unbalanced distribution of moment in the tooth by the inclination of loading force action line, the stress concentration of the tooth root fillet and the sharing the loading force between the concerned tooth and adjacent tooth.

\[
s_t = \frac{W_t K_m 1.0}{F_m J}
\]  

(1)

Where \( W_t \) = tangential force acting on the tooth, N  
\( F \) = face width, mm  
\( m \) = module, mm  
\( J \) = coefficient of geometry  
\( K_m \) = coefficient of dynamic loading  
\( K_m \) = coefficient of load distribution
Design constraint equation is

\[ s_i \leq s = s_s \frac{K_s K_u}{K_c K_x} \]  

(2)

Where \( s_s \) = allowable bending stress

- \( K_s \) = coefficient of application
- \( K_u \) = coefficient of dimension effect
- \( K_c \) = coefficient of life
- \( K_x \) = coefficient of reliability
- \( K_x \) = coefficient of hardness

The results of bending strength designing the speed increaser for high speed machine pump are shown in Table I. The input power and the speed of driving shaft of the machine pump are 37kW and 3550rpm respectively.

**B. Pitting Strength on surface**

The pitting of the tooth surface is regarded as the result of fatigue. The earlier pitting breaks out on the spot where excessive stresses are imposed, and gradually stops as this spot is being worn out or loads are being redistributed over the contact. We used the modified equation of the pitting strength based on Herzian contact theorem not to outbreak sharp pitting within life time as following.

\[ s_s = C_s \sqrt{\frac{W_i C_s C_u C_m C_f}{C_s dF I}} \]  

(3)

where \( C_p \) = coefficient of elastic
d = diameter of pinion pitch
\( F \) = face width, mm
\( I \) = coefficient of geometry
\( W_i \) = tangential force acting on the tooth, N
\( C_a \) = coefficient of application
\( C_v \) = coefficient of dynamic loading

Design constraint equation is

\[ s_i \leq s = s_s \frac{C_s C_u}{C_p C_x} \]  

(4)

Where \( s_s \) = allowable bending stress

- \( C_s \) = coefficient of dimension effect
- \( C_m \) = coefficient of load distribution
- \( C_f \) = coefficient of surface polish
- \( C_l \) = coefficient of life
- \( C_t \) = coefficient of temperature
- \( C_r \) = coefficient of reliability
- \( C_h \) = coefficient of hardness

**TABLE II**

AGMA BENDING STRENGTH

<table>
<thead>
<tr>
<th>Gear factor</th>
<th>Pinion / Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_i (N) )</td>
<td>1027.46</td>
</tr>
<tr>
<td>( K )</td>
<td>0.693465</td>
</tr>
<tr>
<td>( F (mm) )</td>
<td>28.3</td>
</tr>
<tr>
<td>( m (mm) )</td>
<td>1.25</td>
</tr>
<tr>
<td>( K_s )</td>
<td>1</td>
</tr>
<tr>
<td>( K_u )</td>
<td>1.7588 / 1.8523</td>
</tr>
<tr>
<td>( J )</td>
<td>0.41 / 0.49</td>
</tr>
<tr>
<td>( K_l )</td>
<td>0.9</td>
</tr>
<tr>
<td>( K_m )</td>
<td>1</td>
</tr>
<tr>
<td>( K_T )</td>
<td>1</td>
</tr>
<tr>
<td>( K_R )</td>
<td>1</td>
</tr>
<tr>
<td>( S_s (MPa) )</td>
<td>405</td>
</tr>
<tr>
<td>( S_t (MPa) )</td>
<td>179.67 / 158.3</td>
</tr>
<tr>
<td>( S (MPa) )</td>
<td>405</td>
</tr>
</tbody>
</table>

The results of pitting strength on surface designing the speed increaser for high speed machine pump are shown in Table II.
C. Designing Bearing

The input shaft of the vertical centrifugal pump is low speed of 3,550rpm while the output shaft is quite high speed of 17,100rpm. Therefore we can use commercial rolling bearing for input shaft. A pair of deep groove ball bearings 6308 was selected. Estimated life time of this bearing under operating condition was more than 2.39 Mhr which is so enough comparing with the required life time of 4.87×10^4 hr.

In the output shaft, we designed journal bearings with considering the eccentric rate and whirling frequency at high speed, which represent stability of bearing. Usually acceptable values are known as,

\[0.3 \leq \text{eccentric rate} \leq 0.7\]
\[\text{whirling frequency ratio} \leq 0.5\]

Designed journal bearing specification is given in Table III.

Table bearing perturbed from static equilibrium by external force gives stiffness and damping to the shaft through journal. These stiffness and damping is important factor to decide the dynamic characteristics such as the critical speed of global system, the level of vibration and stability, etc.

The model of the stiffness and damping by journal bearing is shown in Figure 1, where the stiffness and damping are functions of rotation speed illustrated in Figure 2.

D. Analysis of Dynamic Characteristics

The speed increaser for high speed machine pump system developing is figured in Figure 3. The power is 37kW, the speed of the input shaft is 3,550rpm, the gear ratio is 1:5 and the speed of output shaft is 17,750rpm. A pair of ball bearings supports the input shaft and journal bearings in the output shaft. Regarding the speed increaser system as lumped system, that consist of the gear chain, shafts, rotors and bearings, we modeled each lumped bodies as 6-DOF system which contains all components of axial, radial, bending, torsion and gyro effect. A pair of spur gear chain is supposed as two rigid bodies with elastic tooth contact, that is a spring element attached two basic circles.

The details of the vibrational model for the spur gear chain with tooth contact are as follows. (1) The equivalent elastic coefficient of the tooth contact is decided when the tooth profile curve on center face of gear goes through the pitch point with consideration of the elastic deformation. (2) The friction at the tooth contact is ignored. (3) The elastic deformation of the tooth just from addendum to dedendum is considered. The deformation of the gear body is not included. (4) The spring coefficient was calculated with consideration of bending and shear deformation by Cornell’s method [10] which assume the gear tooth as cantilever. The deformation of contact is solved by Herzian contact theorem. (5) The power shaft is supposed as a FEM element with mass which stores the kinetic energy and the elastic energy. (6) The rotor is supposed as the element storing the kinetic energy with rigid body motion. (7) Bearings are all supposed as linear springs.
The forcing sources of the speed increaser are due to the unbalance of rotation, the error of gear profile at the tooth contact, the error of gear pitch, the error of assembling bearing into shaft, the change of the tooth stiffness coefficient and the gap or nonlinear deformation at rolling bearing. The main forcing source of gearbox is noted in Table IV.

The vibration and noise of the speed increaser is generated when the forcing frequency coincide with the natural frequency. Noting the forcing frequency as $\Delta_i$, $i = 1, 2, 3, \ldots$ and the natural frequency as $\lambda_j$, $j = 1, 2, 3, \ldots, N$, the resonance occurs when

$$\Delta_i = \lambda_j$$  \hspace{1cm} (5)

Letting $\Delta_i = c_i \omega_i$, the critical speed can be written as

$$\omega_{cr} = \frac{\lambda_i}{c_i}$$  \hspace{1cm} (6)

Where the coefficient $c_i$ is mentioned at Table 4.

The campbell diagram based on Table 4. is shown at Figure 4, where the change of the forcing frequency and the natural frequency is figured with respect to input speed range of 25rpm~4, 00rpm.

![Figure 1 Model of journal bearing](image)

![Figure 2 Effect of rotation speed on stiffness and damping](image)
There are not any critical speed for input unbalance of rotating (1X) nor for output rotating speed (5X) within the range of input operating speed of 3,550rpm. And there is not a critical speed about the forcing component by the gear transmission error (155X) which is included within the noise range.

III. MANUFACTURE AND EXPERIMENT

The speed increaser was manufactured according to aforementioned design which is shown in Figure 5. And the vibration of bearing and shaft were measured as Figure 6. The acceleration sensor is used to pick up the signal of bearing(Figure 6(A)), and the displacement sensor is installed to measure the vibration of the shaft(Figure 6(B)).

The results of each vibrational experiment are illustrated in charts of Figure 7 (A), (B) respectively. We also evaluated the performance and reliability of the prototype based on the API standard strictly. (Table V) The experimental result showed the prototype satisfied all the API standard aspects; vibration of bearing and shaft, noise and lubrication.

IV. CONCLUSION

To development the speed increaser for high speed vertical centrifugal pump, we accomplished gear strength analysis based on AGMA standard, life analysis of rolling bearing, calculation of journal bearing and designing of the speed increaser. Then we manufactured the prototype and evaluated the performance of bearing vibration, shaft vibration and noise. The reliability under strict API standard is verified minutely. As a result, the prototype was corresponded performance demand and reliability.

### TABLE IV

<table>
<thead>
<tr>
<th>Forcing source</th>
<th>Equation</th>
<th>Forcing frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass unbalance $\omega_1$</td>
<td>1X</td>
<td>3,550 rpm</td>
</tr>
<tr>
<td>Mass unbalance $\omega_2$</td>
<td>5X</td>
<td>17,750 rpm</td>
</tr>
<tr>
<td>Gear profile error $\Omega$</td>
<td>155X</td>
<td>550,250 rpm</td>
</tr>
</tbody>
</table>
**Acknowledgment**

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**REFERENCES**


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