Impact of Cladding Permeability on Guided Modes in a Negative-Index Slab Waveguide

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Abstract— We investigate the wave propagation along a slab waveguide composed of a negative index material (Left handed material) and surrounded by conventional media. We predict that such a waveguide can support symmetric modes in such a symmetric structure. We study the dispersion relation of found modes and the associated energy flow. The effect of cladding permeability is studied on dispersion curves and energy flow. We numerically show that the characteristics of modes of the energy flow and the domain of the existence of the mode are significantly changed and can be controlled by varying the permeability of the linear medium.

Keywords—Dispersion relation, Left-handed material, Negative-index material, Energy flow.

I. INTRODUCTION

Metamaterials[1], artificially engineered structures with negative average relative permittivity and permeability, provide a route to creating potential devices with exciting electromagnetic properties that cannot be obtained with natural materials. The field of metamaterials has attracted considerable attention in the scientific community due to the exciting potential applications ranging from perfect lenses [2] and cloaking devices [3,4] to sub-wavelength optical waveguides [5]. Currently, metamaterials (MMs) have been investigated theoretically to determine the normalized frequency effect on electromagnetic wave propagating in MM environment for negative and positive index regime [6]. More linear and nonlinear metamaterials have been investigated theoretically and experimentally from microwave to optical frequencies [7, 8]. Moreover, the left-handed materials (LHMs) are widely used to design novel waveguides systems [9, 10, 11, 12, 13]. It was shown that the properties of the normal guided modes of the metamaterial waveguide are completely different than in a conventional waveguide. For example, Engheta and Alu have shown that there is no cut-off thickness for the first mode of metamaterials waveguide. Thus, this feature provides a solution to the problem of energy transmission with lateral cross section below the diffraction limits [5].

Recent investigations [14, 15, 16, 17] of waves guiding by LHM structures have shown that the properties of guided modes in such systems differ essentially from these of conventional waveguides. In this paper, we derive the dispersion equation and the electric field distributions in a slab waveguide structure, with negative permittivity $\varepsilon_2$ and negative magnetic permeability $\mu_2$ surrounded by conventional material with constant magnetic permeability $\mu_1$ and dielectric permittivity $\varepsilon_1$. To create the left-handed medium, an array of wires has been used, interspersed with an array of split ring resonators [18]. The thin wire array has been shown by Pendry et al [19] to yield an effective dielectric constant of the form $\varepsilon_2(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$, with the plasma frequency $\omega_p$ in the GHz range. The split ring resonator array has been shown by Pendry et al [20] to yield an effective magnetic permeability $\mu_2(\omega) = 1 - \frac{F\omega_0^2}{\omega^2 - \omega_0^2}$, with the resonance frequency $\omega_0$ in the GHz range. The quantity $\omega$ is the angular frequency of the field, $\omega_p$ is the electric plasma frequency, and $\omega_0$ is the effective magnetique plasma frequency, F is the filling factor. This work is based on the study of the action of material parameters and that of the permeability $\mu_1$ on the guided modes, the dispersion curves and the energy flow for cylindrical waveguides, respectively [21, 22, 23]. The paper is organized as follows: in section II, we examined a symmetric waveguide, although we study the dispersion relation, the energy flow, we present the results which presented a new set of behavior, followed by discussions. Section III is devoted to the discussions. In Section IV, we will give the conclusion.
II. ANALYTICAL TREATMENT

A. Theoretical Model

In this paper, Fig. 1 displays the configuration of the proposed waveguide. The waveguide is a symmetric slab waveguide and consists of a core made of a metamaterial having width 2d, surrounded by a linear dielectric [see inset in Fig. 1] with constant magnetic permittivity and dielectric permeability.

We consider a composite structure consisting of negative permittivity and negative permeability, resulting in a negative index of refraction, in the nonlinear wave guide, we study nonlinear guided waves, and we consider a LH (left handed) slab. In this study, we only considered transverse electric field (TE). We look for stationary guided modes in the form:

\[ E = (0, \psi, 0) \exp\left[ j (qx - \omega t) \right] \]

\( \psi(y, z) \) is the electric field. The quantities \( q, \omega \) are the wave vector and the light frequency. For the following analysis, the equation of partial evolution is given by [15], and the field equation in the LHM layer can be written in the form:

\[ \frac{\partial^2 \psi_2}{\partial z^2} + (\varepsilon_2(\omega)\mu_2(\omega) - q^2)\psi_2 + \kappa_2|\psi_2|^2\psi_2 = 0 \quad (1) \]

Where \( \kappa_2 \) is the nonlinearity coefficient.

\( \psi(y, z) \) is the electric field. The quantities \( \varepsilon_2 \) and \( \mu_2 \) are the linear electric permittivity and magnetic permeability, respectively. The quantity \( \kappa_2\mu_2 \) describes the Kerr-type nonlinearity.

Solving Eq. (1) leads to the expressions of 2:

\[ \psi_2 = \pm \frac{2}{\eta_2} \tanh\left( \eta_2(z - z_2) \right) \quad (2) \]

Where \( z_2 \) is a constant.

\[ \eta_2 = \sqrt{\frac{(\varepsilon_2(\omega)\mu_2(\omega) - q^2)}{2}} \]

We assume that \( \kappa_2 < 0 \), and that the dielectric and magnetic responses of the cladding are positives. The field equation in the layer \( i=1,3 \) can be written in the form:

\[ \frac{\partial^2 \psi_i}{\partial z^2} + \chi_i |\psi_i|^2\psi_i = 0 \quad (3) \]
Solving this equation leads to the expressions:
\[
\begin{align*}
\psi_3 &= b_3 e^{\eta_3 (d + z)} & z \leq -d \\
\psi_1 &= b_1 e^{\eta_1 (d - z)} & z \geq d
\end{align*}
\]

\( b_1, b_3, \) are unknown amplitudes.

\( \eta_i^2 = q^2 - \varepsilon_i (\omega) \mu_i (\omega) \geq 0, \) for \( i = \{1, 3\} \)

The boundary conditions at \( z = d \) and \( z = -d \), require the tangentia l components to be continuous.

For the TE waves at the plane \( z = -d \), the boundary conditions read as:

\[
\psi_1 = \psi_2 \\
\frac{1}{\mu_1} \frac{d\psi_1}{dz} = \frac{1}{\mu_2} \frac{d\psi_2}{dz}
\]

Analogous conditions hold for the plane \( z = d \). These conditions determine unknown parameters of the guided wave, namely \( b_1, b_3, z_2 \). In the following, we consider for the symmetric mode:

\( z_2 = 0 \), the corresponding equations can be written in the form:

\[
\begin{align*}
\tanh^{-1} \left[ \frac{b_3}{b_2} \right] &= \eta_2 d \\
\tanh^{-1} \left[ \frac{b_1}{b_3} \right] &= -\eta_2 d \\
\sinh(2\eta_2 d) &= \frac{2\mu_2 \eta_2}{|\mu_2| \sqrt{q^2 - \varepsilon_i \mu_i}}
\end{align*}
\]

Solving equations (2) and (3) with the use of trigonometric identities, we get:

\[
\tanh[2\eta_2 d] = -\frac{2b_1 b_3}{b_2^2 - b_1 b_3}
\]

\[ B. \text{Dispersion Relation for a Nonlinear Layered Structure} \]

From the boundary conditions, we get:

\[
\sinh(2\eta_2 d) \mu_2 \sqrt{q^2 - \varepsilon_i \mu_i} = 2\eta_2 \mu_i
\]

We plot the variation of the normalized frequency. \( \Omega = \omega / \omega_p \) (Fig.2) and the corresponding parameters \( \eta_1 \) and \( \eta_2 \) as a function of the wave number for three values of \( \mu_i \) and (Fig.3), our choice is motivated by the numerical results.

![Figure 2. Dispersion curves of the waveguide, for \( \omega_p = 10GHz; F = 0.56 \), and for different values of \( \mu_i \).](image)

\[ 723 \]
C. Modes of the Energy Flow in the Slab Waveguide

The energy flow in a stationary guided mode has only one component along the waveguide and it can be found as an integral of the pointing vector as [8]:

$$ P_i = \frac{cq}{8\pi\mu_i} \psi_i^2(z) $$

(9)

Where $i = 1; 2; 3$ denotes each of three layers of the structure. We thus obtain for the general case of non-symmetric cladding:

$$ \langle P \rangle = \int_{-d}^{d} P_1 dz + \int_{-d}^{d} P_2 dz + \int_{-\infty}^{\infty} P_3 dz $$

(10)

In the symmetric waveguide, and in the normalized form:

$$ P_i = \left( \frac{\eta_i^2}{\eta_0^2} \frac{\tanh \eta_i d}{\mu_i / \mu_0} \right) \frac{2}{\Omega^2 - \omega^2_0} \left( \eta_i d - \tanh \eta_i d \right) $$

(11)

We give in what follows the evolution of the total energy flow of the LHM layer, we present and discuss the found modes in Fig.4.

III. DISCUSSIONS

The first step of simulation is to refer to the role of the cladding permeability, we plot the variation of the normalized frequency $\Omega = \omega / \omega_p$ as a function of the wave number for three values of $\mu_i$ as presented in Fig.2.
When the wave number \( q \) varies from 1.8 to 2.3, the curves of the normalized frequency are presented between 0.238 and 0.254. Moreover, when \( \mu_i \) is taken from 1 to 2, the zones of variation of the curves are changed and decrease from (0.245 and 0.254) to (0.238 and 0.249) as shown in Fig.2. Thus, one can suggest that the domain of variation becomes larger and larger when \( \mu_i \) increases compared to the result obtained in [24]. The second step of simulations concerns symmetric waveguide and dispersion relation for a nonlinear layered structure, in the aim of giving different trends and emphasizing the new behaviors that can occur when we vary the permeability of the linear medium. Thus, according to the Eq.(12), we lead to the study of the dispersion mode curves as illustrated in Fig.3 which is obtained for, \( \omega_p = 10GHz, \ F = 0.56 \) and \( \ d = 1 \) known as the normalized thickness. Figure3 presents new forms of TE dispersion modes for three values of the permeability \( \mu_i = 1; \ 1.5; \ 2 \), the curves of dispersion modes decrease from 2.1 to 0 as shown in Fig.3. But the domain of variation of that modes becomes significant from (0 and 1.5) to (0 and 2.1) with the increase of \( \mu_i \). Those TE modes become linear for \( \mu_i \) and increase from 1.1 to 2.1 with \( \mu_i \) as presented in Fig.3. Those results are different and new compared to that obtained in [24]. The third step of our analysis corresponds to the energy flow for the modes which is refereed to Eq. (11) and leading to Fig.4 for \( d = 1 \). The curves of energy observed are negatives for the three values of \( \mu_i \) when 1.8 < \( q \) < 2.3. Each of them increases, reaches to zero and begins to decrease as shown in Fig.4, so all the modes practically have similar trajectories. The most important fact is that the variation area of curves of energy becomes narrow from (-125 and -19) to (-40 and -21) when the values of \( \mu_i \) (namely, \( \mu_i = 1; \ 1.5; \ 2 \)) increase. One easily emphasizes that by increasing the quantity \( \mu_i \) the type of these modes changes as the wave number grows. Moreover, the total energy flow \( P_N \) carried by TE modes has a different behavior to that obtained in [24].

The modes are strongly dependent on the parameter \( \mu_i \), so more novel symmetric modes can be supported by the structure and there is a possibility that the energy flow can be controlled by changing the permeability of the linear medium as seen in Fig.4. In addition, we show behaviors and characteristics of the propagating waves which are different to that exhibited by waves propagating in conventional waveguides [25]. Consequently, the influence of cladding permeability on dispersion curves and energy flow leads to the significant change of the domain of existence of modes and the distribution of the waves guided along the left-handed slab.

**IV. CONCLUSIONS**

In this paper, the effect of the permeability of the linear medium in a slab metamaterial waveguide with simultaneously negative permeability \( \mu_2 \) and negative permittivity \( \varepsilon_2 \) has been investigated. For this end, the dispersion relation in a system that consists of a metamaterial film surrounded by a linear cladding is derived, the guided dispersion characteristics of the slab waveguides are numerically investigated for various values of the permeability \( \mu_i \). The energy flow is plotted against the wave number for different values of \( \mu_i \). Firstly, it has been shown that the increase of \( \mu_i \) increases the domain of the existence of the mode, so novel families of guided modes are found in a negative-index film. Secondly, the domain of existence of energy curves become narrow with \( \mu_i \). One of the important finding of this study is the appearance of the backward waves when the normalized energy flow depends on the dimensionless wave number \( q \) and changes as the permeability \( \mu_i \) increase. Finally, we note that the left-handed properties, which required both negative permeability as well as permittivity, thus could not easily be demonstrated using conventional materials is able to reduce the propagation losses, and to increase the mode’s propagation lengths. The results could be applied in theoretical and experimental study of waveguiding structures. These novel modes provide more control over the electromagnetic fields and consequently lead to potential applications in the areas of compact waveguiding structures.
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