Bending Analysis Behaviour of Laminated composite conoidal shell roof using Classical Plate Theory- A Review

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Abstract—Engineering is the emerging technology of the world. In engineering there are various filed out of that civil engineering is very important for the construction of Roads, Bridges, Dams, Canals, Industry, Residential Building And maintenance etc. Civil and structural engineers have always preferred conoidal configuration, which is a ruled and aesthetically pleasant shape. But the variation of curvature is the major difficulty encountered in the analysis of these shells. Keeping above point in mind, a finite element analysis is carried out using an eight noded isoparametric element with five degrees of freedom based on the classical plate theory.

Keywords—laminated composite, conoidal shells, Finite element method and classical plate theory.

I. INTRODUCTION

Different forms of shell structures have been used as roofing units in civil engineering from very early days. The design and use of shell roofs have gradually undergone evolutionary changes with the focus changing with time. For the second half of the twentieth century the finite element technique became very popular as a numerical tool and laminated composites started being applied as structural materials.

Conoidal shell configurations are aesthetically appealing, structurally stiff and they may be used for covering large column free spaces. But the variation of curvature is the major difficulty encountered in the analysis of these shells. The greatest advantage of conoids from the construction point is that they are easy to cast as the surfaces are ruled. These shells are very important industrial roofing units used extensively in the industry. These shells provide uniform lighting to the covered area and are more suitable when greater rise is needed at one end. Aspect ratio and degree of truncation is the major factor leading to variation of curvature which might affect the bending stiffness of such shells.

Hence, in this paper, a study of the bending behaviour of laminated composite conoidal shells is carried out under uniformly distributed pressure with having different stacking sequences by varying aspect ratio and degree of truncation.

II. MATHEMATICAL FORMULATION


A laminated composite conoidal shell as shown in Figure 2.1. The constitutive equations for the shell are given by,

\[ \{ F \} = [ D ] \{ \varepsilon \} \] ---(1)

Where,

\[ D = \begin{bmatrix}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\
A_{12} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} & 0 & 0 \\
A_{16} & A_{26} & A_{66} & B_{61} & B_{62} & B_{66} & 0 & 0 \\
B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\
B_{12} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} & 0 & 0 \\
B_{16} & B_{26} & B_{66} & D_{61} & D_{62} & D_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & S_{11} & S_{12} \\
0 & 0 & 0 & 0 & 0 & 0 & S_{12} & S_{22}
\end{bmatrix} \]

\[ \{ \varepsilon \} = \{ e^x e^y e^0 y e^0 x y k_x k_y k_{xy} e^0 x z e^0 y z \}^T. \] --- (2)
The force and moment resultants are expressed as:

\[
\{F\} = \{N_x \ N_y \ \tau_{xy} \ \sigma_z \ \sigma_xz \ \tau_{xy}z \ \tau_{xz} \ \tau_{yz}\}^T + h/2
\]

\[
\int_{-h/2}^h \{\sigma_{xx} \ \sigma_{yy} \ \tau_{xy} \ \sigma_z \ \tau_{xy} \ \tau_{xz} \ \tau_{yz}\}^T \ dz --- (3)
\]

The stiffness coefficients are defined as:

\[
A_{ij} = \sum_{k=1}^{np} (Q_{i,j,k})(z_k - z_{k-1})
\]

\[
B_{ij} = \frac{1}{2} \sum_{k=1}^{np} (Q_{i,j,k})(z_k^2 - z_{k-1}^2)
\]

\[
D_{ij} = \frac{1}{3} \sum_{k=1}^{np} (Q_{i,j,k})(z_k^3 - z_{k-1}^3)
\]

\[
S_{ij} = \sum_{k=1}^{np} \int F_j (G_{i,j,k})(z_k - z_{k-1})
\]

--- (4)

Where Q_{i,j} are elements of the off-axis elastic constants matrix, which is given by:

\[
[Q_{ij}]_{off} = [T]^T [Q_{ij}]_{on} [T]
\]

--- (5)

In which,

\[
[Q_{ij}]_{on} = \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\]

--- (6)

\[
[T] = \begin{bmatrix}
m^2 & n^2 & mn \\
n^2 & m^2 & -mn \\
-2mn & 2mn & m^2 - n^2
\end{bmatrix}
\]

--- (7)

With, \( m = \cos \theta \) and \( n = \sin \theta \)

The elements of the \([Q_{ij}]_{on}\) matrix are:

\begin{align*}
Q_{11} &= (1-v 12v21)-1 \ E11, \quad Q_{22} = (1-v 12v21)-1 \ E22 \\
Q_{12} &= (1-v 12v21)-1 \ E12 \ v21, \quad Q_{66} = G12
\end{align*}

--- (8)

\( F_i \) and \( F_j \) of equation (4) are two factors presently taken as unity for thin shells, and the elements of the Gij matrix are given by:

\[
G_{xx} = G13 \cos \theta + G23 \sin \theta,
\]

\[
G_{yy} = G13 \cos \theta + G23 \sin \theta,
\]

The strain-displacement relations on the basis of improved first order approximation theory for thin shell are established as:

\[
\begin{bmatrix}
\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}
\end{bmatrix}^T = \begin{bmatrix}
0, 0, 0, 0, 0
\end{bmatrix}^T + \begin{bmatrix}
\kappa_x, \kappa_y, \kappa_{xy}, \kappa_{xz}, \kappa_{yz}
\end{bmatrix}^T
\]

--- (10)

Where the first vector is the mid-surface strain for conoidal shell and the second vector is the change of curvature due to loadings. These are given, respectively, by:

\[
\begin{bmatrix}
\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}
\end{bmatrix}^T = \begin{bmatrix}
\partial u/\partial x, \\
\partial v/\partial y - w/R_y, \\
(\partial^2 u/\partial y^2 + \partial v/\partial x - 2w/R_{xy}, \\
\alpha + \partial w/\partial x, \\
\beta + \partial w/\partial y
\end{bmatrix}
\]

--- (11)

The radius of curvature may be evaluated by differentiating the surface equation of shell in the form \( z = f(x, y) \) and for shallow shells, which are taken up for the present study, the same may be expressed as:

\[
R_{xy} = \frac{1}{\alpha} \frac{\partial^2 z}{\partial x^2} \quad \text{and} \quad \frac{1}{R_y} = \frac{\partial^2 z}{\partial y^2}
\]

--- (12)

III. METHODOLOGY

In this chapter methodology used in the paper is explained. Mathematical formulation using classical plate theory for analysis of plates using Navier’s method is explained.
3.1 Analysis of Plates Using Classical Plate Theory-

Consider a load-free plate, shown in Figure 3.1, in which the x-y plane coincides with the plate’s mid-plane and the z coordinate is perpendicular to it and is directed downwards.

The fundamental assumptions of the linear, elastic, small-deflection theory of bending for thin plates may be stated as follows:

1. The material of the plate is elastic, homogeneous, and isotropic.
2. The plate is initially flat.
3. The deflection (the normal component of the displacement vector) of the mid-plane is small compared with the thickness of the plate. The slope of the deflected surface is therefore very small and the square of the slope is a negligible quantity in comparison with unity.
4. The straight lines, initially normal to the middle plane before bending, remain straight and normal to the middle surface during the deformation, and the length of such elements is not altered. This means that the vertical shear strains γxz and γyz are negligible and the normal strain εz may also be omitted. This assumption is referred to as the “hypothesis of straight normal”.
5. The stress normal to the middle plane, σz is small compared with the other stress components and may be neglected in the stress–strain relations.
6. Since the displacements of a plate are small, it is assumed that the middle surface remains unstrained after bending. Many of these assumptions, known as Kirchhoff’s hypotheses, are analogous to those associated with the simple bending theory of beams. These assumptions result in the reduction of a three-dimensional plate problem to a two-dimensional one. Consequently, the governing plate equation can be derived in a concise and straightforward manner. The plate bending theory based on the above assumptions is referred to as the classical or Kirchhoff’s plate theory.

3.2 Analytical Solution Using Navier’s method-

Navier’s solution by double trigonometric series

Boundary conditions of rectangular plate are as follows:

\[ w = 0 \text{ at } x = 0, x = a \]
\[ w = 0 \text{ at } y = 0, y = b \]
\[ M_x = 0 \text{ at } x = 0, x = a \]
\[ M_y = 0 \text{ at } y = 0, y = b \]
Suppose that the solution of deflection is,

\[ w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

The above solution satisfies the Boundary conditions

The transversal load also can be expanded into double series

\[ q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

Substitution of the above relations into the equilibrium equation gives

Equilibrium equation in terms of the transversal displacement is given below:

\[ \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = q(D) \]

\[ W_{mn} \left( \frac{m^4 \pi^4}{a^4} + 2 \frac{m^2 n^2 \pi^4}{a^2 b^2} + \frac{n^4 \pi^4}{b^4} \right) \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

\[ = \frac{q_{mn}}{D} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

Hence,

\[ w_{mn} = \frac{q_{mn}}{D \pi^4 \left( \frac{m^2}{a^2} + \frac{m^2}{b^2} \right)^2} \]

The final solution is

\[ w(x, y) = \frac{1}{D \pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

**IV. VALIDATION**

In this chapter problems of plate element are analysed using classical plate theory.

Numerical problem of plate element for different aspect ratio

Two problems of Isotropic plate simply supported on all edges are considered by varying aspect ratio.

Dimension and material properties are as follows.

\[ a = b = 2.413 \text{ m}, \ h = 0.5 \text{ m}, \ E = 3.875 \times 10^{10}, \ \nu = 0.15, \ G12 = 1.685 \times 10^{10} \]

**Case I**

Aspect ratio (a/b) = 1

The equilibrium equation in terms of the transversal displacement is given below:

\[ \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} \]

Using Navier’s solution- double Fourier series for plate simply supported on all edges deflection is given as

\[ w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin\left(\frac{m\pi}{a} x\right) \sin\left(\frac{n\pi}{b} y\right) \]

Therefore maximum deflection \( w_{mn} \) =

\[ \frac{1}{D \pi^4} \frac{q_{mn}}{\left( \frac{m^2}{a^2} + \frac{m^2}{b^2} \right)^2} \]

Where,

\[ q_{mn} = \frac{16q_{o}}{mn \pi^2} \] (for uniformly distributed load)

\[ D = \frac{E h^3}{12(1-\nu^2)} \] (for isotropic elements)

\[ q_{mn} = \frac{16 \times 2872.816}{1 \times 1 \times 3.14^2} = 4.662 \times 10^3 \]

\[ D = \frac{3.875 \times 10^{10} \times 0.5^3}{12(1-0.15^2)} = 412.936 \times 10^6 \]

Therefore,

\[ w_{mn} = \frac{1}{412.936 \times 10^6 \times 3.14^2} \frac{4.662 \times 10^3}{\left( \frac{1^2}{2.413^2} + \frac{1^2}{2.413^2} \right)^2} \]

\[ = 9.84 \times 10^{-7} \text{ m} \]
Case II
Aspect ratio (a/b) = 2
The equilibrium equation in terms of the transversal displacement is given below:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

Using Navier’s solution- double Fourier series for plate simply supported on all edges deflection is given as:

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right)$$

Therefore maximum deflection ($w_{mn}$) =

$$w_{mn} = \frac{1}{D} \frac{q_{mn}}{\alpha^4 + \beta^4}$$

Where,

$$q_{mn} = \frac{16q_{0}}{mn \pi^2}$$ (for uniformly distributed load)

$$D = \frac{Eh^3}{12(1-\nu^2)}$$ (for isotropic elements)

$$q_{mn} = \frac{16 \times 2872.816}{1 \times 1 \times 3.14^2} = 4.662 \times 10^3$$

$$D = \frac{3.875 \times 10^4 \times 0.5^3}{12(1-0.15^2)} = 412.936 \times 10^6$$

Therefore,

$$w_{mn} = \frac{1}{412.936 \times 10^6 \times 3.14^4} \frac{4.662 \times 10^3}{(1)^2 + (1)^2}^2$$

$$= 1.574 \times 10^{-7} \text{ m}$$

V. FUTURE SCOPE

There is scope of carrying out similar investigations on delaminated composite conoidal shell roofs for different aspect ratio by varying degree of truncation, different boundary condition and different loading.

REFERENCES


