Detection of a Notch type Damage using Subspace Identification and Artificial Neural Networks

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Abstract — Structural Health Monitoring (SHM) involves the implementation of a damage detection strategy in engineering infrastructures. Introducing a damage alters the modal parameters of the structure. The changes in these parameters can be used to detect damage occurrence. In this work, Subspace Identification (SubID) and Artificial Neural Networks (ANNs) are applied to detect a notch type damage in a thin aluminum plate. The covariance driven output-only SubID was implemented to extract the modal frequencies of the plate. The extracted features are used as the input to a feed-forward ANN for group classification. The ANN was capable of detecting the damage with a success rate of 85%.

Keywords— damage, detection, ANN, subspace, vibration

I. INTRODUCTION

The process of implementing a damage detection strategy in engineering infrastructures is referred to as Structural Health Monitoring (SHM) [1]. SHM gained popularity in the industry for its economic and safety benefits. Reduced downtime and prolonged life cycles of machines generated more revenues. Catastrophic accidents such as bridge failures and airplane crashes due to fatigue cracks can be avoided by early damage detection.

An emerging paradigm of SHM is vibration based damage detection (VBDD). The fundamental principle of VBDD is changes in the physical structure cause a measurable change in the modal parameters (natural frequencies, mode shapes and modal damping) [2]. By measuring these changes, the detection of a damage can be established.

Salawu [3] reviewed the detection of structural damage through changes in natural frequencies. It was concluded that the frequencies obtained from vibrational analysis can be used to monitor the structural condition and may not be sufficient for localization by itself. Natural frequencies have a low damage sensitivity and similar damages at different locations can give the same frequency change [4]. Researchers worked to overcome these disadvantages because modal frequencies can be easily measured with high accuracy. Recent works [4-6] have shown the scope of natural frequencies in damage detection and localization.

From the many identification approaches to identify the modal parameters, subspace identification (SubID) based methods have gained popularity. They have strong capabilities in detecting changes in the eigenstructure of the system [7]. SubID-based methods can handle both input-output and output-only data. Even though input-output methods are more accurate, input data is not always available in practical situations. Investigators have given a lot of attention to this field in recent years.

Baseville [8] developed a non-parametric null-space based residual. It was tested on a two-story frame steel-quake structure. Damage detection was successful, but localization was only partially successful. Saeed et al. [9] used a covariance driven output-only SubID and ANNs to detect and locate a single edge crack on a thin aluminum plate. Damages were detected with a 100% success rate and localization was at 85%. Saeed et al. [10] also implemented the same method on a composite beam structure. Damage detection was positive at 96% and localization at 94%.

Artificial neural networks are models inspired by the central nervous system that map a set of inputs to the desired outputs. Modelling non-linear complex pattern recognition and classification problems are easier with ANNs. The basic idea is to assign class labels to features extracted from the measured data. Researchers continue to use ANNs for all levels of damage detection. Shenoi et al. [11] used a combination of changes in natural frequencies and mode shapes of the vibration signals from an accelerometer and strain gauges. A feed-forward back propagation ANN was implemented to quantify and localize the damage in beam-like structures. Kim et al. [12] used a combination of time-based and modal-based features with ANNs for damage quantification and localization. Omenzetter et al. [13] used finite element models and ANNs to predict a seismic-induced structural damage. The network was able to successfully predict the damage.

II. SUBSPACE IDENTIFICATION

System identification is the process of developing mathematical models of dynamic systems from experimental data.
These models can be used for analyses, simulation, optimization, monitoring, fault detection, prediction, control, etc. The following steps describe the methodology used in this work for feature extraction.

1. Form the covariance matrices from the output data.
2. Form the block Hankel matrix from the covariance matrices.
3. Apply singular value decomposition (SVD) on the block Hankel matrix.
4. Construct the Observability matrix.
5. Equate the Hankel and Observability matrices to obtain the discrete system matrix.
6. Extract the features (natural frequencies) from the discrete system matrix.

### A. State Space Model

A stationary linear dynamic system with \( n_d \) degrees of freedom is described by

\[
M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = u,
\]

\[
y = C_0q,
\]

where \( t \) denotes time and the vectors \( \ddot{q}, \dot{q} \) and \( q \) are the acceleration, velocity, and displacement of \( n_d \) degrees of freedom respectively. \( M \) is the mass matrix, \( D \) is the damping matrix and \( K \) is the stiffness matrix. \( u \) consists of \( n_u \) input forces and the measurement vector \( y \) has \( n_y \) outputs. \( B_0 \) and \( C_0 \) give information about the location of actuators and sensors respectively. \( w \) and \( v \) are the process and noise vectors.

For simplicity, the 2nd order differential equations are converted into state-space form. The discrete state-space model of eqn. (1) and eqn. (2) is given by

\[
x_{k+1} = Ax_k + w_k,
\]

\[
y_k = Cx_k,
\]

where \( x_k \in \mathbb{R}^{n_x}, u_k \in \mathbb{R}^{n_u} \) and \( y_k \in \mathbb{R}^{n_y} \). They are the discrete state-space, input and output vectors respectively at time instant \( k \). \( A \) is the discrete system matrix and \( C \) is the output matrix. The noise vectors \( w_k \) is assumed to be zero-mean Gaussian white noise.

### B. Output-only Subspace Identification

The output covariance is a measure of how two random variables change with respect to each other. It is given by

\[
\Lambda = E[y_{k+1}y_k^T],
\]

where \( E \) is the expectation operator. For a finite number of samples, the covariance matrix is given by

\[
\Lambda_i = \frac{1}{n^i} \sum_{k=1}^{n^i-1} y_{k+i}y_k^T.
\]

### C. Balanced Realization

One of the implementation methods of SubID is balanced realization. The SVD of the Hankel matrix yields

\[
H = \begin{bmatrix}
\Lambda_1 & \Lambda_2 & \cdots & \Lambda_q \\
\Lambda_2 & \Lambda_3 & \cdots & \Lambda_{q+1} \\
\vdots & \vdots & \ddots & \vdots \\
\Lambda_{p+1} & \Lambda_{p+2} & \cdots & \Lambda_{p+q}\end{bmatrix}
\]

Assuming a stationary system with a zero mean white Gaussian noise, the covariance can be rewritten in the form

\[
\Lambda_i = C\Sigma C^T + R \quad i = 0,
\]

\[
\Lambda_i = CA^{i-1}G + R \quad i > 0,
\]

where \( G \) is the cross-covariance between the state and the observed output. This results in the factorization of the Hankel matrix by

\[
H = OC = \begin{bmatrix} C & C A & \cdots & C A^{p-1} \end{bmatrix} \begin{bmatrix} G & A_d G & \cdots & A^{q-1} G \end{bmatrix},
\]

where \( O \) is the Observability matrix of order \( p \) and \( C \) is the Controllability matrix of order \( q \). The rank of \( O \) and \( C \) is \( n_s \).

Where \( \Sigma \) contains the independent column vectors that span the column space of \( H \) and \( V^s \) contains the maximum number of independent row vectors that span the row space of \( H \). The matrix \( U^n \) contains the maximum number of independent column vectors that span the column null space of \( H \) and \( V^n \) contains the maximum number of independent row vectors that span the row null space of \( H \).

The Observability and Hankel matrices have the same left kernel space. Therefore, \( O \) can be written as
The modal (natural) frequencies of the system is derived by
\[
0 = \mathbf{U}^T \mathbf{S}^{1/2} = \begin{bmatrix} C & CA & \cdots & CA^{n-1} \end{bmatrix}.
\] (12)

The modal (natural) frequencies of the system is derived by
\[
f_i = \frac{\omega_i}{2\pi} = \frac{|\ln(v_i)|}{2\pi\tau}, \quad i = 1, 2, \ldots, n_u.
\] (13)

Where \( \ln(v_i) \) are the eigenvalues of \( \mathbf{A} \).

### III. EXPERIMENTATION

#### A. Experimental Setup

A thin aluminium plate of dimensions 480x360x3mm was vertically suspended by a pair of springs. One 3-axis micro electro-mechanical systems (MEMS) accelerometer (LIS331AL) was attached 30mm in the \( x \)-axis and 30mm in the \( y \)-axis from the bottom left of the plate by using a 4 minute epoxy-resin from Hy-Pexy Systems Inc., USA. The accelerometer was micro-soldered onto a printed circuit board by Jupiter Design Technologies Pvt. Ltd., India. The vibrational signals from the \( z \)-axis of the accelerometer was acquired by the NI USB-6211 data acquisition system (DAQ). Post-processing and analyses were conducted on Matlab.

A notch of 50mm length and 5mm wide was machined 200mm from the \( x \)-axis at the bottom of the plate. The plate was randomly excited by a hammer at different locations for a duration of two minutes. Readings were taken from healthy and damaged plates.

#### B. Data Collection & Conditioning

The plate was randomly excited 1000 times to take 500 healthy and 500 damaged samples each. The accelerometer cut-off frequency was set to 500Hz. The sampling frequency of the DAQ was chosen to be 2500 samples/sec, which is more than the Nyquist rate.

The power spectral density (PSD) of the raw signal in figure 2 shows a repetitive noise frequency every 60Hz from the power supply. The peaks at these noise frequencies can interfere during subspace analysis.

A series of band-pass filters were designed on Matlab to filter out the noise. Eight filters were developed to remove the noise at every 60Hz up to the cut-off frequency of 500Hz.

The SubID based feature extraction was applied on the filtered signals. The extracted features are used as the inputs to the ANN for a group classification.

#### C. Group Classification

A group classification is applied on the natural frequencies extracted from the healthy and damaged plates. A multilayer feed-forward perceptron (MLP) was used to classify the frequencies into two categories.
The Matlab neural network toolbox is used for this complex non-linear problem. The ANN is trained by the gradient descent method with an adaptive learning rate. The mean square between the desired output and the network output is the objective function. The ANN is trained with a training database of 400 healthy samples and 400 damaged samples. It was tested with the training and test databases. The test database consists of 100 healthy and 100 damaged samples.

The ANN consists of an input layer, a hidden layer and an output layer. There are 3 input nodes, 5 hidden nodes and 2 nodes in the output layer. Two frequencies ($f_1$ and $f_2$) from the 7th band and one frequency ($f_3$) from the 8th band are the inputs. Three modes from band B7 and B8 are sufficient for analysis purposes as higher frequencies are more sensitive to damage. The output of the ANN indicates the health-status of the plate.

IV. RESULTS

Modes $f_1$, $f_2$ and $f_3$ are analysed for feature selection, extraction and detection. $f_1$ and $f_2$ are in band B7, ranging from 430 to 460Hz. $f_3$ is in band B8, which is from 490 to 520Hz.

A. Feature Selection

Figure 5 shows the PSD of healthy data in B7. The values of $f_1$ and $f_2$ are now 432.2Hz and 451.5Hz. The frequencies of modes $f_1$ and $f_2$ have slightly shifted in the damaged data.

A similar change can be noticed in mode $f_3$ in B8. Figures 7 and 8 show the PSD of healthy and damaged data in B8. The value of $f_3$ is 490.2Hz in the healthy data and 491.5Hz in the damaged data. Since the change in the frequencies in healthy and damaged data can be identified, it can be used for damage detection.
B. Feature Extraction and Classification

The variation of frequencies \( f_1, f_2 \) and \( f_3 \) are shown in figures 9, 10 and 11 respectively. The first five hundred samples on the left side is the healthy data and the second 500 samples are the damaged data. The slight shift in the damaged samples can be clearly seen for all three modes.

Figure 11 shows the feature space of the three modes under investigation. The blue circles represent the healthy data and the red circles represent the damaged data. A mathematical separator should be formed to distinguish the two classes of data. This is done by using the NN toolbox on Matlab.

C. Damage Detection

The ANN was trained with 400 healthy and 400 damaged samples. To test the network, the training database and the test database were utilized. The confusion matrix for the training database is shown in table 1. Out of the 400 healthy training samples tested, 360 samples were undetected and 40 were detected. When 400 damaged training samples were tested, 82 samples were undetected and 318 samples were detected. This gives a damage detection rate of 79.5%, false alarm rate of 5% and a no detection rate of 10.5%.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Confusion matrix of training database</th>
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<tbody>
<tr>
<td>Healthy</td>
<td>Damaged</td>
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<tr>
<td>360</td>
<td>40</td>
</tr>
<tr>
<td>82</td>
<td>318</td>
</tr>
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Multiple plates can be used with different notch locations and sizes to increase the sensitivity of the feature. More data samples can be taken to improve the variation and detection rates.

REFERENCES


V. CONCLUSION

In this work, Subspace Identification (SubID) and Artificial Neural Networks (ANNs) were implemented to detect a notch type damage in a thin aluminum plate. SubID was successfully implemented to select and extract the natural frequencies from the vibration signals. The random excitation of the plate at different locations and noise from the accelerometer caused a variation in the feature (natural frequencies). This made it more difficult to distinguish between the healthy and damaged frequencies. However, the notch type damage was detected with only one sensor by using artificial neural networks.

The ANN successfully classified the data into healthy and damaged groups. The damage detection rate on the experimental data was 85%. The method presented in this work can be implemented to detect predefined damage types in mechanical structures. Natural frequencies can be measured easily and accurately, however they are not the best damage sensitive feature. There was a high variation for a notch type damage of 50mm length and 5mm diameter. It would be even more difficult for a smaller damage.

This work can be continued and there is much scope for future work. Additional placement of sensors can help to measure the mode shapes for localization.

The confusion matrix for the test database is given in table 2. When 100 healthy test samples were passed through the ANN, 87 samples were undetected and 13 samples were detected. For the 100 damaged test samples, 11 samples were undetected and 89 samples were detected. The test database gave a 89% damage detection rate, 6.5% false alarms and a no detection rate of 5.5%.

<table>
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<th>Healthy</th>
<th>Damaged</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>89</td>
<td></td>
</tr>
</tbody>
</table>

By taking the average of the two detection rates, the final damage detection rate is 85%.