Finite Time Control Design for Boiler Feed Water

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Abstract—Finite-time stability is different from traditional stability, which can be mainly shown in two aspects. Firstly, it is research focuses on the state actions of a system in finite time. So, when we study whether a system is finite-time stable, we should give a time interval in advance. Generally, stability research considers a system’s state actions infinite time. Secondly, studying finite-time stability should give a finite state ordinary. We cannot discuss whether a system is finite-time stability simply. This paper considers the problems of finite-time analysis and control design of linear systems and presents the approaches to design the finite-time state feedback controllers of linear systems, and the finite-time output statement feedback controller of linear systems.

Keywords—linear systems; Lyapunov function; finite time stability; state feedback; output feedback

I. INTRODUCTION

In the field of automatic control, stability generally refers to the Lyapunov stability [1,2], which shows the characteristics of the system in the infinite interval. A Lyapunov stable system may hold a bad transient performance, which may have bad influence, even cannot be applied in the field. In order to further understand the transient performance of the system, Peter Dorato considered the problem of short time stability [3] (later called the finite time stability), thus posed a series of the finite time control problem. As the finite time stability is different from the general Lyapunov stability [3,4], more and more attention were paid to the finite time stability [5-7]. The main differences between the finite time stability and the Lyapunov stability are as the following two aspects: Firstly, its research focuses on the state actions of a system in finite time. So, when we study whether a system is finite-time stable, we should give a time interval in advance. Generally, stability research considers a system’s state actions infinite time. Secondly, studying finite-time stability should give a finite state ordinary. We cannot discuss whether a system is finite time stability simply. This paper considers the problems of finite-time analysis and control design of linear systems and presents the approaches to design the finite-time state feedback controllers of linear systems, and the finite-time output feedback controllers of linear systems.

II. THEORY OF FINITE TIME STABILITY

A. Definition of finite time stability

Definition 1. Consider the system as follows [3]:

\[
\dot{x} = f(x), \quad f(0) = 0 \quad (2-1)
\]

Where \( x \in \mathbb{R}^n, f: \mathbb{R}^n \rightarrow \mathbb{R}^n \) is a continuous function. The equilibrium point \( x=0 \) of the system (2.1) is finite time stable if only if the system is stable and convergent in a finite time.

Finite time convergence shows that there exist a continuous function \( T(x) : \mathbb{U} \rightarrow (0, +\infty) \) such that \( \forall x_0 \in \mathbb{U} \subset \mathbb{U} \), the solution of perturbed system is denoted as \( x(t; x_0) \); when \( t \in [0; T(x_0)] \), \( x(t; x_0) \in \mathbb{U} \) and the limit \( \lim_{t \rightarrow T(x_0)} x(t; x_0) = 0 \); when \( t > T(x_0) \), \( x(t; x_0) = 0 \). If \( U = U_0 = \mathbb{R}^n \), we can obtain the concept of continuous finite globally stable.

B. Finite time stability analysis of linear system

The stability criterion of linear system in finite time stability: the foundation is stable is of finite time control system identification a system of continuous finite time. Literature shows that, there are two general methods of verification of continuous finite time control systems: Theory and the homogeneous equation method of finite time Lyapunov stability. Based on some theoretical basis of finite time Lyapunov stability theory and the homogeneous equation method of control system, it is necessary to study some finite time control design problems. The following describes the finite time stability of the two kinds of verification methods, give a scalar function, system of homogeneous equations and vector function is defined.

If the system (3-1), for the global asymptotic stability of the homogeneous degree, and \( k<0 \), the system finite time is global stable. In addition, reference [8,9] gives Lyapunov stability criterion of finite time control system.
Assume that there is continuously differentiable function $V: U \rightarrow R$, satisfying the following conditions\[(6,7)\]

(1) $V$ is positive function.

(2) There exist positive real number $c > 0$ and $\alpha \in (0; 1)$, and a open neighborhood $U_0 \subseteq U$ including the origin. And the following inequality

$$
\dot{V}(x) + cV^\alpha \leq 0, x \in U_0
$$

holds.

Then the system (3-1) is finite time stable; If $U = U_0 = R^n$, then the system (2-1) is finite time globally stable.

### III. FINITE TIME STATE FEEDBACK DESIGN

Consider the state equations as follows \[(6)\]

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Gw(t), x(0) = x_0 \\
\dot{w}(t) &= Fw(t) \\
y(t) &= Cx(t) + Dw(t)
\end{align*}
$$

Where vector $x(t) \in R^n$ is state, $u(t) \in R^n$ is the control input, $o(t) \in R^n$ is the unknown disturbance input system, initial value $o(t) = o_0$ is unknown but satisfies $o_0^T o_0 \leq \delta^2$; where $\delta$ is positive constant, $y(t) \in R^n$ is the measurable output of system, $A \in R^{n \times n}, B \in R^{n \times m}, C \in R^{m \times n}, D \in R^{m \times m}, F \in R^{n \times n}, G \in R^{n \times m}$ are known constant matrix.

In the system (3-1), when $o(t) = 0$

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bx(t) + Gw(t), x(0) = x_0 \\
y(t) &= Cx(t) + Dw(t)
\end{align*}
$$

Where matrix is definite d as that in (3-1).

When $y(t) = 0$,

$$
\begin{align*}
\dot{x}(t) &= Ax(t) + Bx(t) + Gw(t), x(0) = x_0 \\
\dot{o}(t) &= Fw(t)
\end{align*}
$$

For system (3-2), setting $c_1, c_2, \delta, T$, where $c_1 < c_2$, positive matrix $R \in R^{n \times n}$, if there exist non negative constant $\alpha$, matrix $L \in R^{p \times n}$ and two positive definite matrices $Q_1 \in R^{n \times n}$, $Q_2 \in R^{k \times k}$ satisfying

$$
\begin{pmatrix}
AQ_1 + BL + Q_1A^T + U^T B - \alpha Q_1 & G \\
G^T & F^T Q_1 + Q_2 - \alpha Q_2
\end{pmatrix} < 0
$$

(3-4)

$$
\frac{C}{\lambda_{max}(Q_2)} + \lambda_{max}(Q_1) \delta < C e^{-\alpha T}
$$

(3-5)

Where $\tilde{Q}_1 = R^{1/2} Q_1 R^{1/2}$, then we can design a state feedback controller $u = Kx(t)$. $K = LQ_1^{1/2}$ such that the closed loop system (3-3) finite time bounded with respect to $(c_1, c_2, \delta, T, R)$.

### IV. FINITE TIME STATE FEEDBACK DESIGN

First, we assume that (3-1) the state of the system can be used, in the same way, use method 3.1 to find the (3-4) system in finite time $K$ matrix, then the design can make the system (3-4) output feedback control design state can make the system still has a finite time boundedness properties in the $K$ matrix has been under the. At present, we assume that the (3-2) $K$ matrix has been, and still can make the system state output feedback controller with finite time boundedness.

For system (3-2), given $c_1, c_2, \delta, T$ are positive number, and $c_1 < c_2$, $R \in R^{n \times n}$ is positive definite and symmetric. If $\alpha \geq 0$, $M_1 \in R^{n \times n}$, $N \in R^{n \times n}$, $M_2 \in R^{n \times n}$ are positive and symmetrical matrix, $S \in R^{m \times n}$, we can obtain matrix

$$
\begin{pmatrix}
\sum_{i=1}^{11} - M_1 BK & M_1 G \\
- K^T B^T M_1 & \sum_{i=22}^{22} M_2 G - SD \\
G^T M_1 & G^T M_2 - D^T S^T \\
F^T N - NF - \alpha N
\end{pmatrix} < 0
$$

(4-1)

$$
\begin{pmatrix}
\lambda_{max}(\tilde{M}_1) + \lambda_{max}(\tilde{M}_2) \delta < \lambda_{max}(\tilde{M}_1) c_1 e^{-\alpha T}
\end{pmatrix}
$$

(4-2)
where
\[ \sum_{i=1} = A^T M_1 + BK^T - \alpha M_1 \]
\[ \sum_{22} = A^T M_2 - C^T S + M_2 A - SC - \alpha M_2 \]
\[ \tilde{M}_1 = R^{-\frac{1}{2}} M_1 R^{-\frac{1}{2}} \]
\[ \tilde{M}_2 = R^{-\frac{1}{2}} \tilde{M}_2 R^{-\frac{1}{2}} \]

Then, we can obtain the output feedback controller:
\[ \dot{x}(t) = A\dot{x}(t) + Bu(t) + (Hy - C\dot{x}(t)) \cdot \dot{x}(t) = 0 \quad (4-3) \]
\[ u = K\dot{x}(t) \quad (4-4) \]

Such that the system is finite time bounded with respect to \( 0(c_1, c_2, \delta, T, R) \) and \( H = M_2^{-1} S \).

V. DESIGN FOR BOILER FEED WATER

A. Boiler feed water control system

Boiler feed water boiler automatic control system is one of the main control units. The main task of this system is to keep the water level equal to the given value. Automatic control systems of boiler feed water flow drum are single stage three element feed water flow automatic control system and the three element cascade feed water flow of two kinds of control schemes of automatic control system [8,9].

Drum water level is an important monitoring parameter of drum boiler, it indirectly reflects the balance, the water level between the boiler steam load and water is too high, the steam water phenomenon, influencing normal production of steam unit. Drum water level is too low, will affect the boiler steam drum water level of natural circulation, maintain normal are necessary to ensure the safe operation of boiler and steam turbine.

Many factors affect the drum water level change, mainly coal consumption, water flow and main steam flow rate etc. Because of the change of the amount of coal on the water level of the impact is small, so it is easy to adjust. Therefore, we generally only considering the influence of main steam flow and water flow can be generated on the water. Boiler water level regulation principle structure is shown in Figure 1.

![Figure 1: Feed water control structure diagram](image)

1-feed water tube; 2-valve; 3-combustor; 4-drum; 5-pipe; 6-steam superheater; 7-tube

The water level in the water flow under the change from the following transfer function:
\[ W_b(s) = \frac{H}{W} = \frac{e}{s - \frac{\varepsilon}{1 + \varepsilon} = \frac{e}{s(1 + \varepsilon)}} \quad (5-1) \]

B. Finite control design and simulation

The transfer function is given for drum water level flow
\[ W(s) = \frac{Y(s)}{U(s)} = \frac{0.037}{s(1 + 30s)} \quad (5-2) \]

By the system (4-2) system transfer function matrix can be obtained directly system:
\[ A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad C = (0.037 \ 0) \]

Based on the finite time control design in section 4 we can obtain
\[ G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & -1 \\ 2 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 \\ -20 \end{pmatrix} \]

\( c_1 = 1, c_2 = 26, \delta = 0, T = 2 \) definite matrix \( R = I \), where \( I \) is the two-dimensional unit matrix.

Using the MATLAB LMI control toolbox, we can compute and obtain \( K = [0.62 \ -17.07] \).

Initial conditions are \( x_0 = [-0.6 \ 0.8] \) and \( \omega_0 = [3.8 \ \ 1.6] \), where \( \alpha = 1, \beta = 2 \), matrix \( L_4, Q_4 \) are
\[ L_4 = (10.97 \ -11.54), \quad Q_4 = \begin{pmatrix} 1.76 & -0.46 \\ -0.46 & 0.54 \end{pmatrix} \]
Based on the above conditions, we can obtain

\[ H = \begin{pmatrix} 22.36 \\ -2.26 \end{pmatrix} \]

Selecting \( \alpha = 1 \), \( M_2 \), \( S \) as

\[ M_2 = \begin{pmatrix} 32.88 & 16.37 \\ 16.37 & 143.45 \end{pmatrix}, \quad S = \begin{pmatrix} 14.38 \\ -176.24 \end{pmatrix} \]

The simulation results in Figure 2 and Figure 3, the simulation results show that, state feedback and output feedback finite time can the drum water level control, state feedback k control effect better than output feedback.

**VI. DESIGN FOR BOILER FEED WATER**

According to the stability theory of linear system, this paper introduced the finite time, based on the finite time control design method. Including state feedback finite time control and design of finite time observer based output feedback. The simulation examples verify the feasibility and effectiveness of this algorithm. And the finite time control method is applied to the flow control system of boiler feed water, design of the finite time drum level controller, the simulation results achieve the finite time stability of the system show that the designed controller, to achieve the expected goal. The proposed method can well solve the finite time control problem for the linear systems, and can be extended to nonlinear system, has important significance for theory research and engineering application.

**Acknowledgment**

This work was in part supported by the National Natural Science Foundation of China (Grant No.61304019, Grant No. 61304126), the Hunan Provincial Natural Science Foundation of China (Grant No.61304019, Grant No.12C0004). The authors, hereby, gratefully acknowledge their support.

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