Synthesis Algorithm for Optimal Control of Multidimensional Discrete Dynamic Object

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Abstract - The problems of the synthesis of optimal control algorithm focused on the synthesis of control actions, modulated in amplitude and allows to transfer control to a given steady state in the shortest possible time and ensure a stable and sustainable presence in the local area. An efficient algorithm for finding the control actions based on the use of prediction error values.

Keywords – algorithm, optimal control, synthesis, prediction error values, control action, state variables, discrete transfer function, regulator.

I. INTRODUCTION

At the present time the control of multi-dimensional discrete dynamical objects developed a large number of different methods and schemes [1, 3, 5]. However, their use is complicated by the large labor intensive and cumbersome computational procedures and also the need to perform various kinds of simplifying assumptions and conditions [6]. For example, using compensation controls are often obtained as a result of synthesis controller structure is physically unrealizable. Analogous results can be obtained using various methods isolation channels multidimensional object of control with using auxiliary cross regulators [3]. Multivariate polynomial aperiodic regulators [2] often give unacceptable results-based of control, as obtained by the value of the overshoot and unworkable for physical reasons, the calculated values of the control actions.

In this article describes the algorithm developed by the authors of optimal control, focused on the synthesis of discrete control actions, amplitude-modulated, and allows the calculation of the value of the control pulses with a constant period, transforming the object into a new control required, the steady state, and provides a stable and sustainable locate object in his the surrounding area.

It is known that the basis for the synthesis of the basic algorithm of the control algorithm is necessary position that after the required number $N$ of cycles control, the state of an object must satisfy the conditions:

$$y_i(N_u) = Z_i, \quad i = 1, N,$$

$$y_i^k(N_u) = 0, \quad i = 1, N_u - 1. \quad (1)$$

There $N_u$ - the number of output variables of the control object;

$y_i(N_u)$ - values of the $i$-th output variable at the end of $N_u$-th cycle;

$y_i^k(N_u)$ - the value of the $k$-th derivative of the $i$-th output variable;

$Z_i$ - the desired value (setpoint) $i$-th output variable.

Minimum number of cycles for the control of multidimensional control object is defined by the formula:

$$N_u = \max \left\{ \sum_{j=1}^{M} n_{ij} \right\}, \quad j = 1, M,$$

there $M$ – the number of input variables of the control object (CO);

$n_{ij}$ – the order of the transfer function of the channel $j$-th input - $i$-th output.

Conditions (1), in other words, can be interpreted as follows:

$$y_i(N_u) = Z_i, \quad i = 1, N,$$

$$y_i^k(N_u + j) = 0, \quad i = 1, N_u - 1$$

i.e. required at the end of $N_u$ control measures necessary to achieve the desired reference variable and then for $N_u$ measures the output variable has to be at a given level.

In contrast to the base, the proposed algorithm synthesis, mathematical formulation of the control problem is the following:

$$y_i(L + N_u) = E_i(L + N_u) = Z_i - y_i^k(L + N_u), \quad i = 1, N,$$

$$U_k(L + j) = \text{const}, \quad j = 1, N_u - 1, \quad k = 1, M. \quad (2)$$
In the last equation: \( L \) - current moment of time (clock quantization); \( M \) - the number of input control actions; 
\( y_l^j(L+N_u) \) - the predicted value of the \( i \)-th input variable at \( N_u \) cycles ahead of the current time; \( E \) - error signal in the \( L+N_u \) control cycle.

Analysis of the conditions (2) leads to the conclusion that the implementation of this algorithm, the control system doesn’t seek to ensure that the control object after the selected number of cycles \( N_u \) control was transferred to the new steady state characterized by constant and unchanged values of the output variables. The algorithm only allows the calculation of the values of the control actions, based on the condition that the output variables at the end of \( N_u \) cycles control their required values. In the current cycle of quantization is assumed that the control action during the next control interval will not change [3]. In fact, in each new cycle shall be recalculated values of control actions for each input variable. This approach provides a stable, asymptotic motion control object to a new steady state and stable finding variables that characterize its behavior in a given range of control. Such a smooth variation of the output variables is carried out due to the fact that in each subsequent cycle amplitude of the control pulses under conditions of constant desired values (setpoints) and minor fluctuations of conditions of changing a relatively small amount, which does not cause any sudden movements of the object and control system.

In condition (2) to determine the values of \( y_l^j(L+N_u) \) can be used, by the expression:

\[
y_l^j(L+N_u) = \sum_{j=1}^{M} \sum_{l=0}^{N_u-1} U_j(l) \cdot \omega_{lj}(N_u-l), \quad i=1, N; \quad k = 0, N_u-1.
\]

There \( U_j(l) \) - desired value of \( j \)-th control in the \( l \)-th cycle; 
\( \omega_{lj} \) - the value of the \( k \)-th derivative of the weight function of the channel \( j \)-th input - \( i \)-th output in the \( n \)-th cycle.

Moreover, the weight function is defined as a response to single pulse duration equal to the value tact of quantization

\[
k \cdot \omega_{lj}(n) = h_{lj}(n) = h_{lj}(n) - h_{lj}(n-1)
\]

There \( h_{lj}(n) \) - the value of the \( k \)-th derivative of the transition function of a channel \( j \)-th input - \( i \)-th output in the \( n \)-th cycle quantizing.

Finding \( u_j^l(l), \quad j=1,M \) should be preceded by determination of the values of the error vector \( E \) in accordance with the actual state of the control object. This is due to the fact that the actual, the state of the control object by virtue of a mathematical model inaccuracies, errors in the implementation of the calculated control actions, the availability of various disturbing factors acting on the real object, almost never coincides with the state, calculated only on the models.

To determine the predicted values of the \( E \)-vector, on the assumption that during the next control interval control actions don’t change, you can use the model of the control object in the space of state variables:

\[
\begin{align*}
X(N_u \times T_0) &= \phi(N_u \times T_0) \times X(N_u \times T_0) \\
Y(N_u \times T_0) &= C \times X(N_u \times T_0)
\end{align*}
\]

where: \( X(N_u \times T_0) \) - a state vector value of the control object at the \( t = N_u \times T_0 \) time’s moment; 
\( \phi(N_u \times T_0) \) - value of fundamental (transition) matrix control object for the value of the \( t = N_u \times T_0 \) argument; 
\( C \) - input matrix; 
\( Y \) - vector of output variables of the control object.

To implement the expression (3) in the prediction of output variables of the object required the number of cycles required, firstly, to use another kind of model - a model of the control object represented in the space of state variables, and secondly to determine the values of the state variables from the known values of the output variables necessary to use an observer status, for example Kalman filter. These two circumstances significantly increase unwieldiness of the algorithm and entail additional errors.

Therefore, to determine the predicted values of the error, or what is the same, the predicted values of the output variables, an approach based on the use of discrete transfer functions of the control object [2, 4, 5]. Let us denote by:

\[
W_j(z) = \frac{B_j(z)}{A_j(z)}
\]

Discrete channel transfer function \( j \)-th input - \( i \)-th output, where \( B_j(z) \) and \( A_j(z) \) - polynomials in powers of degrees \( n_{ij}^+ \) and \( n_{ij}^- \). Then the Z-transform of the output variable can be represented as:

\[
y_j(z) = \sum_{j=1}^{M} \frac{B_j(z)}{A_j(z)} \times u_j(z)
\]
Citing the right side of expression (4) to a common denominator, obtain:

\[ y_j(z) = \sum_{i=1}^{M} A_j(z) \sum_{k=1}^{M} B_j(z) \prod_{k \neq j} A_k(z) u_j(z) \]  

or in another form:

\[ y_j(z) = \sum_{i=1}^{M} D_j(z) u_j(z) \]  

(5)

where

\[ D_j(z) = B_j(z) \prod_{k=1}^{M} A_k(z) \]

Dividing both sides of (5) to \( c_i(0) \cdot z^{-m} \), obtain

\[ y_i(z) = R_i(z^{-1}) \]  

(6)

Where, \( R_i(z^{-1}) \) and \( D_i(z^{-1}) \) - polynomials in inverse powers of Z form:

\[ R_i(z^{-1}) = 1 + \sum_{k=1}^{n_i} p_i(k) z^{-k} \]

\[ D_i(z^{-1}) = 1 + \sum_{k=0}^{n_i} r_i(k) z^{-k} \]  

(7)

where: \( r_i(k) = \frac{c_i(k)}{c_i(0)} \) \( p_i(k) \frac{d_i(k)}{c_i(0)} \)

On the basis of expression (6) with (7) is easily obtained recursion formula for predicting the output values of the control object:

\[ y_j(L+1) = \sum_{k=1}^{n_j} \eta_j(k) \times y_j(L+1-k) + \sum_{k=1}^{n_j} \sum_{j=1}^{M} p_j(k) \times u_j(L-k+1) \]  

(8)

Here \( n_j \) - the total order of the transfer functions associated with the \( i \)-th output variable. After spending the procedure (8) \( N_u \) times and considering that and \( u_j(L) = u_j(L+1) = \ldots = u_j(L+N_u-I) = \text{const} \), can obtain an expression for the calculation of the predicted value of the output variable in the \((L+N_u)\)-th cycle.

To get the required number of predicted values of the output variables, the formula (8) must be used \((N_u + n_i)\) for each \( i \)-th output variable.

Thus advantage of formula (8) is the possibility of use in predicting the actual values of the output variables. To illustrate this fact, the formula (8) can be written as:

\[ y_j(L+1) = \sum_{k=1}^{n_j} \eta_j(k) y_j(L+1-k) - \sum_{k=1}^{n_j} \eta_j(k) y_j(L+1-k) + \sum_{j=1}^{M} \sum_{k=0}^{n_j} p_j(k) u_j(L+1-k) + S_j(L) \]

(9)

Where, \( S_j(L) \) - actual and estimated (predicted) values of the \( i \)-th output variable in the \((L+1)\)-th cycle that; \( S_j(L) \)-term containing the unknown to the current time value of the control actions: \( u_j(L), u_j(L+1), \ldots, u_j(L+m_j) \).

For the input variable associated with the output variables with at least one non-static transmission channel, these values are assumed to be zero, and the corresponding terms \( S_i(L) \) will also be zero. If the order of transmission channels astatizy any input variable is equal to zero, it is necessary to take:

\[ u_j(L) = u_j(L+1) = \ldots = u_j(L+2 \cdot N_u) = u_j(L) \]

That is, it is assumed that since the current time values of all control actions on the considered input received equal value, providing the desired steady state of control object, and that this assumption is calculated predicted values of the output variables.

The main drawback of the approach is to first establish a steady state values of the variables. And to solve this problem it is necessary each time changing the desired output vector of variables, which leads to an increase in the number of transitions of the proposed algorithm. A method based on the use of derivatives of the weighting functions does not have this drawback, but it is inherent in its more significant and unavoidable difficulties computing.

Combine the advantages and eliminate the disadvantages of these two approaches can be, if the formation of a system of linear equations using the equation

\[ y_j(L+K) = \sum_{j=1}^{M} \Delta u_j(L) \cdot h_j(L+K-l) \]  

(9)

Where, \( \Delta u_j(L) \) - increment \( j \)-th input control to the \((L-K)\)-th cycle relative to its full value for \((L-1)\)-th step time, i.e.:

\[ \Delta u_j(l) = u_j(l) - u_j(l-1) \]

And using the equation (9) for forming a system of linear equations leads to the necessity of increasing the dimension of the latter by the amount M due to the fact that, for each input variable is automatically incremented by a number of unknown parameters characterizing the control pulse sequence.
In this case work the control interval remains the same, that is equal to $N_u$ cycles. The last full meaning of the control action:

$$u_j(k_i + m_j) = u_j(k_i - 1) + \Delta u_j(k_i + k)$$

automatically it turns equal to the value that provides the desired steady state of the control object.

The direct use of the basic algorithm for the control multi-dimensional object, especially in cases where its transmission channels have a pronounced asymmetry in gain and dynamics of the processes can lead to unacceptable results. This can be expressed as in getting unacceptably large quantities of overshoot in the transition from the initial state to the desired final steady state in the minimum possible number of cycles control and to obtain unrecoverable due to technical reasons the amount of control. In the case of a one-dimensional object of control is usually transferred to the use of aperiodic regulators increased dimensionality that, in principle, have a clearly does not guarantee acceptable results control. When using such regulators simply assumed that the initial control action must be equal to its maximum possible value with the appropriate sign. For multidimensional object control, this approach clearly is unacceptable because it is often not possible to determine not only the magnitude but also the sign of the initial control action.

In the proposed scheme to address these deficiencies is proposed to increase the amount of space control over the minimum required number of cycles to control any of the output variables or for all. But this increase is due to the inclusion of additional control actions, make transfers of property is the initial state from which the problem to be solved at a lower level control system satisfies at least one of the following quality criterion:

$$I_1 = \min \left\{ \sum_{k \in M} \sum_{i=1}^{m_k^d + m_k^r} a_k(i) \right\}$$

$$I_2 = \min \left\{ \sum_{k \in M} \sum_{i=1}^{m_k^d + N_u} \beta_k \cdot e_k^2(i) \right\}$$

where, $N_i$ – set of indices of the output variables, the sum of squared errors for which it is necessary to minimize;

$\beta_k(i)$ – weighting factor takes into account the importance of k-th square error of the output variable in the i-th cycle;

$e_k(i)$ – error value on the k-th output variable of the i-th cycle.

The use of other types of criteria, in particular, a minimum sum of squares of increments of control actions or combinations of criteria (10) and (11).

For solving optimization problems (10), (11), or the need to express similar functional to be minimized through the unknown, written by control actions. In this case the functional is obtained as the square of the added control actions. Minimization of the functional operating elementary method of least squares.

II. CONCLUSIONS.

1. To solve the problem control process multidimensional object proposed two-level scheme for the synthesis of discrete control actions.
2. Solution of the lower level allows you to transfer control from an arbitrary initial state to a desired final state is steady in the minimum possible number of cycles controls and ensure a stable and sustainable finding an object in a given areals.
3. By increasing the number of cycles on the upper control level optimization problem is solved, which allows to transfer the object with the minimum value of the selected criterion of quality control.

REFERENCES