Effect of Quantum Cooperation in Three Entangled Ants

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Abstract—Recently, the physical concept of quantum computation model has inspired the information and computer science domain. Summhammer applied the physical concept of quantum entanglement to cooperative behavior of two ants/agents pushing a pebble which may be too heavy for one ant. According to his results, we have confirmed that the two quantum-inspired ants imitating quantum entanglement state, i.e., two entangled ants, can push the pebble up to twice relative to the two classical ants in independent relation, i.e., two independent ants. In the previous study, the quantum entanglement state of only two qubits, which is called Bell state, has been applied to cooperative behavior. In this study, we novelty applied the quantum entanglement state of three qubits, which is called GHZ (Greenberger-Horne-Zeilinger) state, to cooperative behavior, and performed its simulation. From the experimental analysis, we have confirmed that the three entangled ants can push a pebble up to thrice relative to the three independent ants.

Keywords—agents, cooperative forces, GHZ (Greenberger-Horne-Zeilinger) state, quantum cooperation, quantum entanglement

I. INTRODUCTION

In recent years, the physical concept of quantum computation model has inspired the information and computer science domain. For instance, the physical concept of quantum interference and Quantum superposition were innovated to Genetic Algorithm [1] and Evolutionary Algorithm [2], respectively. And furthermore, Summhammer applied the physical concept of quantum entanglement to cooperative behavior of two ants/agents pushing a pebble which may be too heavy for one ant [3]. That is, each of ants makes measurements on quantum states to decide whether to execute certain actions. In Summhammer’s model, the ants make odour-guided random choices of possible directions, followed by a quantum decision whether to push or to rest. According to his results, we have confirmed that the two quantum-inspired ants imitating quantum entanglement state, i.e., two entangled ants, can push the pebble up to twice relative to the two classical ants in independent relation, i.e., two independent ants. Then, Nakayama et al. have simulated the cooperation of two ants while changing the condition of important parameters and have clarified its feature [4, 5].

From the experimental analysis, in competitive society where ants with strong force are advantageous, Nakayama et al. have proven that two homogeneous ant brains are good and two heterogeneous ant forces are good. The result of the experimental analysis was similar to the idea in collective decision making. In the previous studies described above, the quantum entanglement state of only two qubits, which is called Bell state, has been applied to cooperative behavior. In this study, we novelty applied the quantum entanglement state of three qubits, which is called GHZ (Greenberger-Horne-Zeilinger) state, to cooperative behavior, and performed its simulation. This paper describes the experimental results.

II. OVERVIEW OF QUANTUM ENTANGLEMENT IN THE PHYSICAL DOMAIN USED IN THREE ENTANGLED ANTS

In this paper we deal with the simplest kind of quantum entanglement. It is the correlation of the angular momentum between three particles of the same kind. The angular momenta of three such particles can easily be measured along different directions. The possible results are then \( |\uparrow\downarrow\downarrow\rangle \), \( |\downarrow\uparrow\downarrow\rangle \), \( |\downarrow\downarrow\uparrow\rangle \), and \( |\uparrow\uparrow\downarrow\rangle \), where \( |\uparrow\rangle \) and \( |\downarrow\rangle \) are spin up and spin down, respectively. Quantum theory can only predict the probabilities, \( p_{\uparrow\uparrow\downarrow} \), \( p_{\downarrow\uparrow\downarrow} \), \( p_{\downarrow\downarrow\uparrow} \), and \( p_{\uparrow\downarrow\downarrow} \), for these measurement results. An important state of quantum entanglement, which will be used in ants, is the so-called GHZ state. Symbolically, this state is written as

\[
|\psi_{GHZ}\rangle = \frac{|\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\downarrow\rangle}{\sqrt{2}}.
\]  

III. NOVEL MODEL OF THREE ANTS PUSHING A PEBBLE

We assume that three ants must push a pebble towards a certain goal. Each ant ant\(j\) \((j = 1, 2, 3)\) can push with a force \(f_j\). In order to move the pebble, a minimum force \(f_{min}\) must be applied. Clearly, if the pebble is too heavy to be moved by any two of the ants, the three ants must push simultaneously, and they must push in similar directions.
The three ants achieve their task by making a series of simultaneous push attempts. A push attempt is successful if the force applied to the pebble is larger than the required minimum. Then the pebble will move a little path length proportional to the force in the direction of the force.

Before a push attempt, three ants must make two decisions as follows. These decisions are made independently by each ant and they are not communicated to the other ants.

1st Decision Choose each direction \( \theta_j \) that each ant \( ant_j \) pushes. The direction \( \theta_j \) is chosen in accordance with a probability distribution \( \omega(\theta_j) \) as shown in

\[
\omega(\theta_j) = n(\pi - z|\theta_j|),
\]

where \( n \) is an appropriate normalization factor and \( z \) is a positive constant of \([0,1]\).

2nd Decision Decide either to really push at this attempt or to have a little rest. These decision processes of three entangled ants and three independent ants are described in Section IV. B. and IV. C., respectively.

If the ant \( ant_j \) decides to push, the force \( f_j \) applied to a pebble is given as follows:

\[
f_j = s_j \begin{bmatrix} \sin \theta_j \\ \cos \theta_j \end{bmatrix}.
\]

Where \( s_j \) is the strength of the ant, that is, equals to \(|f_j|\). The direction \( \theta_j \) is \(-\pi \leq \theta_j < \pi\), where \( \theta_j = 0 \) is the direction straight to the goal. For example, \( f_1, f_2 \), and \( f_3 \), which are the forces of \( ant_1, ant_2, \) and \( ant_3 \), are depicted in Fig. 1.

IV. Decision Process Of Whether Each Ant Really Pushes A Pebble At This Attempt Or Has A Little Rest

A. Real Space of a Force and Brain Space of a Qubit in a Quantum-Inspired Ant

In general, a qubit is described by two-dimensional column vector in the complex vector space where the inner product is defined. It uses the following computational basis states \(|\uparrow\rangle\) and \(|\downarrow\rangle\) as orthonormal base vectors.

\[
|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
\]

The qubit can have a stochastic superposition state (vector sum) of the two vectors \(|\uparrow\rangle\) and \(|\downarrow\rangle\) with each complex probability amplitude. The superposition state \(|\psi\rangle\) of the qubit can be shown as follows:

\[
|\psi\rangle = \alpha |\uparrow\rangle + \beta |\downarrow\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}.
\]

Where \( \alpha \) and \( \beta \) are the complex probability amplitudes to observe the state of \(|\uparrow\rangle\) or \(|\downarrow\rangle\), respectively. They are normalized as \(|\alpha|^2 + |\beta|^2 = 1\). \(|\alpha|^2\) is the probability that the state of \(|\uparrow\rangle\) is observed, and \(|\beta|^2\) is the probability that the state of \(|\downarrow\rangle\) is observed. In this paper, the observation result \(\uparrow\rangle\) corresponds to pushing a pebble, and the observation result \(\downarrow\rangle\) corresponds to resting without pushing a pebble.

Next, let us think the relation between “the force direction to push a pebble in real space of a force” and “the behavior (pushing a pebble or resting) of a quantum-inspired ant \( ant_q \) in brain space of a qubit” by using a single qubit. As shown in Fig. 2(a), let \( \theta \) \((-\pi \leq \theta < \pi\) be the angle between the goal direction and the force direction to push a pebble. The \( \theta \) is the parameter relating to behavioral decision of a quantum-inspired ant \( ant_q \) which either pushes a pebble or rests. The ratio of the probability amplitudes in the superposition state \(|\psi\rangle\) changes depending on the \( \theta \), and its change is performed by unitary transformation. To perform the unitary transformation, the following rotation matrix \( R \) can be used. That is, the ratio of the probability amplitudes is changed by rotating a qubit depending on the force direction.

\[
|\psi'\rangle = R |\psi\rangle = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix} |\psi\rangle.
\]
For instance, the rotation of the initial state $\ket{\uparrow}$ is given as follows:

$$
R_j\ket{\uparrow} = \begin{bmatrix}
\cos(\theta_j/2) & -\sin(\theta_j/2) \\
\sin(\theta_j/2) & \cos(\theta_j/2)
\end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \ket{\uparrow}
= \cos(\theta_j/2) \ket{\uparrow} + \sin(\theta_j/2) \ket{\downarrow},
$$

(7)

and Fig. 2(b) shows the rotation result. That is to say, the probability to push a pebble becomes higher if $|\theta_j|$ becomes smaller, and the probability to push a pebble becomes lower if $|\theta_j|$ becomes larger. Thus, the real space of a force is associated with the brain space of a qubit.

### B. Cooperative Behavior of Three Entangled Ants

The probability whether to push a pebble or to rest at each attempt, this is the place where we permitted the physical concept of quantum entanglement to come in, just like Summhammer. Cooperative behavior occurs to three quantum-inspired ants, which have the above-mentioned behavioral decision process, by imitating quantum entanglement state. Here, the decision process of the cooperative behavior is explained. First, we think three qubits state $|\psi_{\text{GHZ}}\rangle = (|\uparrow_1\uparrow_2\uparrow_3\rangle + |\downarrow_1\downarrow_2\downarrow_3\rangle)/\sqrt{2}$ called GHZ state in which three ants $\text{ant}_{q_1}$, $\text{ant}_{q_2}$, and $\text{ant}_{q_3}$ are entangled. The force directions to push a pebble of three quantum-inspired ants $\text{ant}_{q_1}$, $\text{ant}_{q_2}$, and $\text{ant}_{q_3}$ are $\theta_1$, $\theta_2$, and $\theta_3$, respectively. The rotation matrices corresponding to $\theta_1$, $\theta_2$, and $\theta_3$ are $R_{\theta_1}$, $R_{\theta_2}$, and $R_{\theta_3}$, respectively, where $-\pi \leq \theta_1, \theta_2, \theta_3 < \pi$. Then, the state $|\psi_{\text{GHZ}}\rangle$ is translated by the linear operator $R_{\theta_1} \otimes R_{\theta_2} \otimes R_{\theta_3}$ as follows:

$$
|\psi_{\text{GHZ}}\rangle \rightarrow R_{\theta_1} \otimes R_{\theta_2} \otimes R_{\theta_3} (|\uparrow_1\uparrow_2\uparrow_3\rangle + |\downarrow_1\downarrow_2\downarrow_3\rangle)/\sqrt{2}
$$

$$
= \frac{1}{\sqrt{2}} \left( R_{\theta_1} |\uparrow_1\rangle \otimes R_{\theta_2} |\uparrow_2\rangle \otimes R_{\theta_3} |\uparrow_3\rangle + R_{\theta_1} |\downarrow_1\rangle \otimes R_{\theta_2} |\downarrow_2\rangle \otimes R_{\theta_3} |\downarrow_3\rangle \right)
$$

where

$$
R_{\theta} |\uparrow\rangle = \cos\frac{\theta}{2} |\uparrow\rangle + \sin\frac{\theta}{2} |\downarrow\rangle
$$

and

$$
R_{\theta} |\downarrow\rangle = \cos\frac{\theta}{2} |\downarrow\rangle + \sin\frac{\theta}{2} |\uparrow\rangle
$$

(8)

Figure 2: Relation between force direction to push a pebble in real space of a force and behavior of a quantum-inspired ant $\text{ant}_q$ in brain space of a qubit.
Therefore, the probability to observe $|t_1t_2t_3\rangle$ in which $ant_{q1}$, $ant_{q2}$, and $ant_{q3}$ cooperatively push a pebble is given as follows:

$$p_{1213} = \frac{1}{2} \left[ \cos \left( \frac{\theta_1}{2} \right) \cos \left( \frac{\theta_2}{2} \right) \cos \left( \frac{\theta_3}{2} \right) - \sin \left( \frac{\theta_1}{2} \right) \sin \left( \frac{\theta_2}{2} \right) \sin \left( \frac{\theta_3}{2} \right) \right]^2.$$  

Next, the probability $p_{1t_2t_13}$ to observe $|t_1t_2t_13\rangle$ in which only $ant_{q1}$ and $ant_{q2}$ cooperatively push a pebble is given as follows:

$$p_{1t_2t_13} = \frac{1}{2} \left[ \cos \left( \frac{\theta_1}{2} \right) \cos \left( \frac{\theta_2}{2} \right) \sin \left( \frac{\theta_3}{2} \right) + \sin \left( \frac{\theta_1}{2} \right) \sin \left( \frac{\theta_2}{2} \right) \cos \left( \frac{\theta_3}{2} \right) \right]^2.$$  

Similarly, other six probabilities, $p_{1t_1t_23}$, $p_{1t_2t_13}$, $p_{1t_1t_23}$, $p_{1t_1t_23}$, $p_{1t_1t_23}$, and $p_{1t_1t_23}$, are also given.

C. Behavior of Three Independent Ants

The decision process of the behavior in three independent ants is very simple. In three classical ants in independent relation, $ant_{c1}$, $ant_{c2}$, and $ant_{c3}$, each ant independently decides either to push a pebble or to rest in accordance with the probability $1/2$. As a result, the eight probabilities $p_{1t_2t_3}$ in three independent ants are given as follows:

$$p_{1t_2t_3} = \frac{1}{8},$$  

where the mark * is $\uparrow$ or $\downarrow$.

Incidentally, the expected displacement of a pebble after one push attempt in three independent ants is same as that of three entangled ants.

V. Basic Experiment in Quantum Cooperation of Three Entangled Ants

In order to confirm the effects of cooperative behavior in three entangled ants, we have performed the numerical simulation while changing the important parameter $f_{min}$. We changed $f_{min}$ from 0.01 to $3 \times s_j$ in every +0.01 steps, where $s_j$ is the strength of $ant_j$. The other parameter values used are shown in Table I. The number of push attempts in one trial, $t_{max}$, was 1,000 and we analyzed the performance based on the average values of 10 trials. The experimental result is shown in Fig. 3. The figure shows the mean distance pushed by ants in a straight line between a starting point and a last point of a pebble as a function of the minimum force necessary to push a pebble, $f_{min}$, in the case of $s_j = 0.50$. Incidentally, the cases of $s_j = 0.30$ and 0.66 have also shown the same tendency as $s_j = 0.50$.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Value used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pebbles, $n_p$ [pebbles]</td>
<td>1</td>
</tr>
<tr>
<td>Number of ants, $n_a$ [ants]</td>
<td>3</td>
</tr>
<tr>
<td>Strength of $ant_j$, $s_j$</td>
<td>0.30, 0.50, 0.66</td>
</tr>
<tr>
<td>Moving distance of a pebble [dots/strength]</td>
<td>4</td>
</tr>
<tr>
<td>Positive constant in Eq. (2), $z$</td>
<td>2/3</td>
</tr>
</tbody>
</table>
Figure 3: Experimental result in the case of $s_f = 0.50$.

From the experimental analysis, we have confirmed that the three quantum-inspired ants imitating quantum entanglement state, i.e., three entangled ants, can push a pebble up to thrice relative to the three classical ants in independent relation, i.e., three independent ants.

VI. CONCLUSIONS

We have novelly applied the quantum entanglement state of three qubits, which is called GHZ (Greenberger-Horne-Zeilinger) state, to cooperative behavior of three ants pushing a pebble which may be too heavy for one ant or two ants. From the experimental analysis, we have confirmed that the three entangled ants can push a pebble up to thrice relative to the three independent ants.

In the near future, we plan to analyze the cooperative behavior of three entangled ants in more detail. Finally, we expect that these results can be utilized for studying the optimal modeling of cooperative relations in society by means of a simulation and designing algorithms of games, etc.

REFERENCES