Application of FAHP to Debris-Flow Hazards Risk Assessment Using Generalized Exponential Fuzzy Numbers

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Abstract—This paper presents the fuzzy analytical hierarchy process (FAHP) using generalized exponential fuzzy numbers employed for risk assessment of debris-flow occurrence. Three layers are involved in the structure of FAHP, the goal layer, criteria layer and the sub-criteria layer. In the criteria layer and sub-criteria layer we consider nine major influence factors grouped into three categories: (1) topological and geological conditions including slope angle, type of deposit, grain size distribution, and surface plants; (2) watershed conditions including effective watershed area and quantity of outflow of sediment; (3) rainfall conditions including intensity, duration, and accumulated rainfall. The relative judgment among each influence factor is built up based upon 9-scale to form the fuzzy reciprocal judgment matrices for evaluating weighting vectors for each layer. Generalized exponential membership functions are employed for fuzzy numbers concerning vagueness and uncertainty of human judgment. Two cases of debris-flow disasters occurred Taiwan are tested by the use of FAHP: one is the debris-flow occurred in Sen-Mu Village, Nan-Tou County, July 31, 1996 and the other is the debris-flow occurred in Hua-San Village, Yun-Lin County, September 17-18, 2001. The results show that the proposed FAHP model using generalized exponential fuzzy numbers as well as the associated influence factors and criteria can successfully predict the risk of debris-flow hazard occurrence. Furthermore, the predicted overall risk indices obtained from FAHP using generalized exponential fuzzy numbers are smaller than those obtained from FAHP using triangular and trapezoidal fuzzy numbers and AHP. When wider generalized exponential fuzzy numbers are employed, the predicted values are smaller.

Keywords—Debris-flow, Fuzzy Analytical Hierarchy Process (FAHP), Multi-criteria, Risk Assessment, Generalized Exponential Fuzzy Numbers.

I. INTRODUCTION

Natural disasters caused by debris flows often occur on earth recently and there are many natural or manmade factors leading to these tremendous accidents (Takahashi, 1991) [1]. Debris flow is a special type of hyper-concentrated flow, composed of mud, clay, sand, gravels, water, air and so forth, flowing down mainly due to its gravitational force.

Development of prevention techniques is obviously based on the understanding and analysis of the mechanical behavior of debris flow. However, nowadays prediction and risk assessment of the occurrence of the debris flow hazards becomes more and more important for many countries.

Occurrence of debris flow depends highly on local topographic, meteorologic, geologic, and hydrologic conditions. Many disasters caused by debris-flows and muddy flows in Taiwan were reported and studied (Jan and Shen, 1993; Wu et al., 2006; Lin, 2006) [2-4]. However, the special reasons that Taiwan is prone to debris-flow hazards can be summarized as follows (Jan, 2000; Huang, 2001) [5, 6].

We know that risk assessment task is a multi-level and multi-criteria complicated process. The method based on probability theory requires a lot of statistical data, prior probability, model evidence, and likelihood function, are required for validity of Bayesian inference. In many real cases and especially for debris-flow hazards this is very difficult.

In the field of operational research, Saaty, T. L. (1980) had proposed the so-called Analytic Hierarchy Process (AHP) for multiple criteria decision making problems [7]. It is also a good approach for risk assessment of problem with multi-criteria on influence factors. The author also had attempted applied AHP to risk assessment of debris-flow hazards occurred in Sen-Mu and Hua-San [8] and Tung-Men and Tung-Shing [9], respectively, and the results are verified to be useful.

Considering vagueness involved in life and engineering problems Zadeh (1965) [10] first introduced the theory of fuzzy sets. Then some researchers tried to combine fuzzy logic with analytic hierarchy process, termed Fuzzy Analytic Hierarchy Process (FAHP), and applied to decision making, safety or risk assessment for commercial and engineering practices. The author has conducted preliminary study on the application of FAHP to debris-flow hazards with case studies using triangular fuzzy numbers (Huang, 2015) [11], and using trapezoidal fuzzy numbers (Huang, 2015) [12], respectively.
The results show that the ranking of importance of influence factors to overall risk obtained from AHP and FAHP models are the same. Accumulated rainfall (AR), duration of rainfall (DR), and averaged slope angle (SA) play the first three leading important influence factors of debris-flow hazards. Both two kinds of FAHP using trapezoidal and triangular fuzzy numbers predict smaller overall risk indices than the AHP model which use crisp judgment matrices. On the other hand, FAHP using trapezoidal fuzzy numbers yields smaller overall risk indices than FAHP using triangular fuzzy numbers and the reason is due to the wider vagueness involved in the trapezoidal fuzzy numbers than the triangular fuzzy numbers.

However, the most frequently tested membership functions in FAHP such as triangular and trapezoidal ones are linear functions which describe the vagueness increasing and decreasing from the centered value. We might wonder what the prediction will be if the nonlinearity of membership functions is involved. Normal probability distribution (Gaussian distribution) is of course can be tested but its parameters are only two (the mean and the standard deviation). Here another nonlinear but with character of similar exponential type membership functions, the so-called generalized exponential functions [13, 14], will be employed for risk assessment of debris-flow hazards. Three layers and nine influence factors (criteria) are involved in the structure of FAHP. These nine major influence factors grouped into three categories: (1) topological and geological conditions: critical slope, type of deposit, grain size distribution, surface plants; (2) effective watershed area: area of watershed, outflow of sediment; (3) rainfall conditions: intensity, duration, accumulated rainfall. The relative judgment among each influence factor is built up based upon 9-level to form the reciprocal judgment matrices for evaluating weighting vectors for each layer. Generalized exponential fuzzy numbers are employed for accounting for the vagueness and uncertainty of human judgment. Cases of disasters occurred in Sen-Mu Village of Nan-Tou County and Hua-San Village of Yun-Lin County, respectively. Furthermore, effect of different vagueness of generalized exponential functions on the predicted values will be studied.

II. FAHP USING GENERALIZED EXPONENTIAL FUZZY NUMBERS

2.1 Establish a hierarchical model

The top of a hierarchical model is the goal to be achieved by FAHP. The second layer (criteria layer) refers to the proposed major categories of influence factors.

Factors in the third layer (sub-criteria-layer) support the factors in the second layer.

2.2 Setup comparison matrices

In general, comparison matrices are obtained through filling in a questionnaire form by some experts in this field. Here we adopt 1-9 scaling method as suggested by Saaty (1980) [27] as shown in Table 1 and Figure 1. Here we establish the following judgment matrices:

$$[\tilde{A}] = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix} \tag{1}$$

where each entry is a generalized exponential fuzzy number defined as

$$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}) \tag{2}$$

and described as a generalized exponential membership function as [14]

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 0, & x \leq a_{ij} \\ \exp(-\frac{b_{ij} - x}{b_{ij} - a_{ij}}), & a_{ij} \leq x \leq b_{ij} \\ 1, & b_{ij} \leq x \leq c_{ij} \\ \exp(-\frac{x - c_{ij}}{d_{ij} - c_{ij}}), & c_{ij} \leq x \leq d_{ij} \\ 0, & d_{ij} \leq x \end{cases}$$

which can be adjusted to satisfy the reciprocal relationship:

$$\tilde{a}_{ji} = 1/\tilde{a}_{ij} \tag{4}$$

The example of definition of a fuzzy number

$$\tilde{a} = \frac{1}{3} = (a, b, c, d) = (1.5, 2.5, 3.5, 4.5)$$

and its inverse

$$\tilde{a}^{-1} = \frac{1}{3} = (\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}) = (1, 0.41, 0.33, 0.25, 0.15)$$

is shown in Figure 2.

As compared with those judgment matrices employed in AHP, the entries of the judgment matrices employed in FAHP are fuzzy numbers with which the vagueness of human consideration and assessment can be included based on the theory of fuzzy sets. It should be noticed that there are many different forms of fuzzy judgment matrices, in some usages the entry for equal judgment is 0.5.
intermediate risk 2.3 Calculate the fuzzy relative weights

The weights for each influence factors in each layers can be calculated using root method as follows:

\[
\tilde{Z}_i = (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \cdots \otimes \tilde{a}_{in})^{1/n}, \quad i = 1, 2, \ldots, n
\] (5)

\[
\tilde{W}_i = \tilde{Z}_1 \otimes (\tilde{Z}_2 \otimes \cdots \otimes \tilde{Z}_n)^{-1}, \quad i = 1, 2, \ldots, n
\] (6)

Where \( \otimes \) and \( \oplus \) denotes the addition and product of fuzzy numbers and obeys the rule of operation, respectively:

\[
\tilde{a}_1 \oplus \tilde{a}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2),
\]

\[
\tilde{a}_1 \otimes \tilde{a}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2),
\]

\[
\tilde{a}_1^{-1} = (d_1^{-1}, c_1^{-1}, b_1^{-1}, a_1^{-1})
\]

\[
\tilde{a}_1^{1/n} = (a_1^{1/n}, b_1^{1/n}, c_1^{1/n}, d_1^{1/n})
\] (7)

2.4 Defuzzify the relative weights

In the practical application the relative weights can be transformed into crisp values for convenient usage. In addition, the calculation of relative weights does not involve vague human thought and fuzzy logic no need considered. There are a lot of approaches for ranking (sorting) fuzzy number and can be classified into four types as reported by Chen & Hwang (1992) [15].

There also exist many modified and extended methods for ranking fuzzy numbers based on the previous list approaches. However, for the current usage of generalized exponential membership functions the centroid method is the same as the fuzzy mean and can be calculated as
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\[
\frac{\int_{l_i}^{u_i} x \mu_{w_i}(x) \, dx}{\int_{l_i}^{u_i} \mu_{w_i}(x) \, dx} = \frac{Q_i}{A} \quad (8a)
\]

with

\[
Q_i = e^{-1}[(b-a)(b-2a+ae^1)+(d-c)(c-2d+de^1)] + \left(c^2 - b^2\right)/2
\]

\[
A = (1-e^{-1})(d-c+b-a)+(c-b)
\]

From which we can calculate the crisp relative weights for each level,

2.5 Nomalize the relative weights

The relative weights for each influence factor in both layers are not normalized and can be done as

\[
w^*_i = \frac{w_i}{\sum_{i=1}^{n} w_i}, \quad i = 1,2,\ldots,n
\]

Which satisfies

\[
\sum_{i=1}^{n} w^*_i = 1, \quad i = 1,2,\ldots,n
\]

2.6 Evaluate the final assessed risk index

Calculate the relative risk impact (RRI) of each influence factor on the overall risk:

\[
[RRI]_{C \rightarrow A} = [M]_{C \rightarrow B} \bullet \{w^*_i\}_{B \rightarrow A} \quad (11)
\]

Where \([RRI]\) is a \(n \times 1\) crisp vector denoting the relative risk impact factor for each influence factor (C layer) on the global risk (A layer); \([M]\) is a \(n \times m\) crisp transfer matrix that converting each sub-criteria-layer (C layer) to the criteria-layer (B layer) and \(\{w^*_i\}\) is a \(m \times 1\) crisp transfer vector that converting criteria-layer (B layer) to goal layer (A layer).

Once the RRI of a FAHP (or AHP) model is obtained we can plot the bar chart to observe the relative importance of each sub-criteria-layer influence factor on the overall risk and the assessment model has been built up for practical assessment application.

2.7 Built up the evaluation criteria for each influence factor:

When we want to make the assessment of a hazard, we first investigate the data of the case based on the influence factor from well-established evaluation criteria and assign the evaluation values \(\bar{E}_i (i = 1,2,\ldots,n)\) where the fuzzy numbers are also considered to include the vagueness of human judgment.

\[
\bar{E}_i = (E^i_a, E^i_b, E^i_c, E^i_d) (i = 1,2,\ldots,n)
\]

The evaluation values can be assigned based on the judgment way as depicted in Table I.

2.8 Calculate the overall risk index (ORI):

Then we can calculate the fuzzy overall risk index (ORI):

\[
\bar{\Gamma} = (\Gamma_a, \Gamma_b, \Gamma_c, \Gamma_d) = [RRI]^T \bullet \{\bar{E}\} = \sum_{i=1}^{n} RRI_i \bullet \bar{E}_i
\]

Using the method of centroid for defuzzification of generalized exponential fuzzy number the final crisp global risk index is

\[
ORI = \Gamma = \frac{\Gamma_a}{\Gamma_A}
\]

With

\[
\Gamma_a = e^{-1}[(\Gamma_b - \Gamma_a)(\Gamma_d - 2\Gamma_a + \Gamma_d) + (\Gamma_a - \Gamma_d)] + (\Gamma_c - \Gamma_d)/2
\]

\[
\Gamma_A = (1-e^{-1})(\Gamma_d - \Gamma_c + \Gamma_b - \Gamma_a) + (\Gamma_c - \Gamma_b)
\]

III. RISK ASSESSMENT OF DEBRIS-FLOW OCCURRENCE - FAHP

3.1 Establish a hierarchical model

The top of a hierarchical model is the goal to be achieved by AHP. In this research that is the risk assessment of debris-flow hazards. The second layer (criteria layer) refers to the proposed three major categories of influence factors: (B1) Topological and Geological Conditions; (B2) Effective watershed area conditions; and (B3) Rainfall conditions. Factors in the third layer (sub-criteria-layer) support the factors in the second layer and are denoted from \(C_{11}, C_{12}, \ldots, C_{33}\) as shown in Figure 3.
3.2 Setup comparison matrices

Here the following judgment matrices were built up as follows:

\[
[B] = \begin{bmatrix}
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1)
\end{bmatrix}
\]

\[
[C_1] = \begin{bmatrix}
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1)
\end{bmatrix}
\]

\[
[C_2] = \begin{bmatrix}
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1)
\end{bmatrix}
\]

\[
[C_3] = \begin{bmatrix}
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1) \\
(1,1,1) & (1,1,1) & (1,1,1)
\end{bmatrix}
\]

As compared with those judgment matrices employed in AHP (Huang, 2014 [7]), the entries of the judgment matrices employed in FAHP are fuzzy numbers with which the vagueness of human consideration and assessment can be included based on the theory of fuzzy sets.

3.3 Calculate the relative weights and the maximal eigenvalues

\[
\tilde{w}_{B1} = (0.66, 0.84, 1.19, 1.52) \otimes (\frac{1}{4.94}, \frac{1}{4.00}, \frac{1}{3.03}, \frac{1}{2.39})
\]

\[
= (0.13, 0.21, 0.39, 0.64)
\]

\[
\tilde{w}_{B2} = (0.37, 0.45, 0.58, 0.74) \otimes (\frac{1}{4.94}, \frac{1}{4.00}, \frac{1}{3.03}, \frac{1}{2.39})
\]

\[
= (0.08, 0.11, 0.19, 0.31)
\]

\[
\tilde{w}_{B3} = (1.36, 1.74, 2.24, 2.68) \otimes (\frac{1}{4.94}, \frac{1}{4.00}, \frac{1}{3.03}, \frac{1}{2.39})
\]

\[
= (0.28, 0.43, 0.74, 1.12)
\]

\[
\tilde{w}_{C11} = (139, 1.90, 2.51, 3.05) \otimes (\frac{1}{6.76}, \frac{1}{5.35}, \frac{1}{3.97}, \frac{1}{3.03})
\]

\[
= (0.21, 0.36, 0.63, 1.01)
\]

\[
\tilde{w}_{C12} = (0.73, 0.93, 1.31, 1.68) \otimes (\frac{1}{6.76}, \frac{1}{5.35}, \frac{1}{3.97}, \frac{1}{3.03})
\]

\[
= (0.11, 0.17, 0.33, 0.55)
\]

\[
\tilde{w}_{C13} = (0.56, 0.70, 0.95, 1.24) \otimes (\frac{1}{6.76}, \frac{1}{5.35}, \frac{1}{3.97}, \frac{1}{3.03})
\]

\[
= (0.08, 0.13, 0.24, 0.41)
\]

\[
\tilde{w}_{C14} = (0.35, 0.43, 0.60, 0.80) \otimes (\frac{1}{6.76}, \frac{1}{5.35}, \frac{1}{3.97}, \frac{1}{3.03})
\]

\[
= (0.05, 0.08, 0.15, 0.26)
\]

\[
\tilde{w}_{C21} = (1.22, 1.58, 1.87, 2.12) \otimes (\frac{1}{2.94}, \frac{1}{2.50}, \frac{1}{2.12}, \frac{1}{1.70})
\]

\[
= (0.42, 0.63, 0.88, 1.25)
\]

\[
\tilde{w}_{C22} = (0.47, 0.53, 0.63, 0.82) \otimes (\frac{1}{2.94}, \frac{1}{2.50}, \frac{1}{2.12}, \frac{1}{1.70})
\]

\[
= (0.16, 0.21, 0.30, 0.48)
\]

\[
\tilde{w}_{C31} = (0.40, 0.49, 0.64, 0.87) \otimes (\frac{1}{4.56}, \frac{1}{3.73}, \frac{1}{2.90}, \frac{1}{2.31})
\]

\[
= (0.09, 0.13, 0.22, 0.38)
\]

\[
\tilde{w}_{C32} = (0.76, 0.95, 1.26, 1.52) \otimes (\frac{1}{4.56}, \frac{1}{3.73}, \frac{1}{2.90}, \frac{1}{2.31})
\]

\[
= (0.17, 0.25, 0.43, 0.66)
\]

\[
\tilde{w}_{C33} = (1.14, 1.46, 1.83, 2.16) \otimes (\frac{1}{4.56}, \frac{1}{3.73}, \frac{1}{2.90}, \frac{1}{2.31})
\]

\[
= (0.25, 0.39, 0.63, 0.94)
\]
3.4 Defuzzify the relative weights

Using Eq. (8) we can calculate

\[ w_B^1 = \frac{\left( w_B^1 \right)_A}{\left( w_B^1 \right)_A} \times \frac{0.139293}{0.3839} = 0.3628 \]

Similarly we can calculate \( w_B^2, w_B^3, \ldots, w_C^33 \).

3.5 Normalize the relative weights

After normalization using Eq. (9) we have

\[ w_B^* = 0.3628/1.2138 = 0.2989 \]
\[ w_B^* = 0.1815/1.2138 = 0.1495 \]
\[ w_B^* = 0.6695/1.2138 = 0.5515 \]
\[ w_C^1 = 0.5571/1.2635 = 0.4567 \]
\[ w_C^2 = 0.3100/1.2635 = 0.2453 \]
\[ w_C^3 = 0.2300/1.2635 = 0.1820 \]
\[ w_C^4 = 0.1465/1.2635 = 0.1160 \]
\[ w_C^5 = 0.8150/1.1206 = 0.7274 \]
\[ w_C^6 = 0.3055/1.1206 = 0.2726 \]
\[ w_C^7 = 0.2189/1.1870 = 0.1844 \]
\[ w_C^8 = 0.3950/1.1870 = 0.3328 \]
\[ w_C^9 = 0.5732/1.1870 = 0.4829 \]

3.6 Evaluate the final assessed risk index

The calculated results are:

\[ \{ RRI \} = [M]_{C \rightarrow B} \bullet \{ w_B^* \}_{B \rightarrow A} \]

\[ \begin{bmatrix} 0.4567 & 0 & 0 \\ 0.2453 & 0 & 0 \\ 0.1820 & 0 & 0 \\ 0.1160 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.2989 \\ 0.1495 \\ 0.5515 \end{bmatrix} \]

\[ = \begin{bmatrix} 0.0437 \\ 0.1088 \\ 0.0408 \\ 0.1017 \\ 0.1835 \\ 0.2663 \end{bmatrix} \]

And are plotted in Figure 4 and Table II where the previous AHP results [8], FAHP using triangular fuzzy numbers [11], and FAHP using trapezoidal fuzzy numbers [12] are also shown together for comparison. Only a small deviation exists between AHP and two FAHP results. Furthermore, the ranking of importance of influence factors to overall risk obtained from AHP and two FAHPs models are kept the same.

Figure 3 Hierarchical model of the risk assessment of debris-flows hazards using AHP and FAHP
3.7 Built up the evaluation criteria for each influence factor:

The evaluation value and criteria for debris-flow occurred in Taiwan is proposed as reported in Ref. [8, 11-12] and not repeated here.

IV. CASE STUDY OF RISK ASSESSMENT OF DEBRIS-FLOW HAZARD

4.1 Description of Debris-Flow Disasters Occurred in Taiwan

(1) Case 1: Debris-Flow Occurred in Sen-Mu Village

The first case of debris-flow disaster we employed for checking the validity of risk assessment model is that occurred at July 31, 1996, in the Sen-Mu Village, Shin-Yi township, Nan-Tou County, Taiwan during the attack of Typhoon with strong storms. This debris-flow caused 27 deaths, 17 persons disappeared. A lot of valuable investigation data were collected and reported by Yu (1997)[16], Su (1997) [17], Chen et al. (1999) [18], Chen (2000) [19], and Chen et al. (2010) [20].

(2) Case 2: Debris-Flow Occurred in Hua-Shan Village

The second debris-flow hazard occurred in September 17, 18, 2001 at Hua-Shan Village, Gu-Ken township, Yun-Lin County, Taiwan during the attack of Typhoon. A lot of gravels and stones moved down from Hua-Shan River to the village destroyed some houses. In fact at June 3, 2000 debris-flow had occurred once in this village due to thunder storm and large amount of stones moved from Ker-Giau River and caused disasters. Many field investigation data were collected and reported by Chen (2006) [21] and Tsai (2007) [22].

4.2 Recorded data and the judgment vector {E}

The recorded data of debris-flows occurred at Sen-Mu Village and Hua-San Village can be observed in Ref. [8, 11-12], respectively.

4.3 FAHP Risk Assessment Results

From Eq. (14), we can calculate the overall risk index of the debris-flow occurred in Sen-Mu Village and Hua-San Village, respectively, as

$$\tilde{\Gamma} = \sum_{i=1}^{a} RRI_i \cdot E_i = (6.5203, 7.5203, 8.1698, 8.4687)$$

$$ORI = \Gamma = \frac{Q_1}{A} = \frac{11.1819}{1.4705} = 7.5836$$

Figure 4 Diagram for relative risk impact (RRI) for influence factors of debris-flow hazards using AHP and three FAHPS

<table>
<thead>
<tr>
<th>INFLUENCE FACTORS</th>
<th>RRI % (AHP)</th>
<th>RRI % (FAHP) (Triangular)</th>
<th>RRI % (FAHP) (Trapezoidal)</th>
<th>RRI % (FAHP) (Present)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>C11 (SA)</td>
<td>13.64</td>
<td>13.75</td>
<td>13.66</td>
<td>13.65</td>
<td>3</td>
</tr>
<tr>
<td>C12 (TD)</td>
<td>6.82</td>
<td>7.30</td>
<td>7.26</td>
<td>7.33</td>
<td>6</td>
</tr>
<tr>
<td>C13 (SD)</td>
<td>5.03</td>
<td>5.31</td>
<td>5.38</td>
<td>5.44</td>
<td>7</td>
</tr>
<tr>
<td>C14 (SP)</td>
<td>3.08</td>
<td>3.50</td>
<td>3.42</td>
<td>3.47</td>
<td>9</td>
</tr>
<tr>
<td>C21 (EA)</td>
<td>10.71</td>
<td>11.12</td>
<td>10.87</td>
<td>10.88</td>
<td>4</td>
</tr>
<tr>
<td>C22 (OS)</td>
<td>3.57</td>
<td>3.86</td>
<td>3.99</td>
<td>4.08</td>
<td>8</td>
</tr>
<tr>
<td>C31 (IR)</td>
<td>9.52</td>
<td>9.87</td>
<td>10.07</td>
<td>10.17</td>
<td>5</td>
</tr>
<tr>
<td>C32 (DR)</td>
<td>19.04</td>
<td>18.45</td>
<td>18.45</td>
<td>18.35</td>
<td>2</td>
</tr>
<tr>
<td>C33 (AR)</td>
<td>28.57</td>
<td>26.85</td>
<td>26.90</td>
<td>26.63</td>
<td>1</td>
</tr>
<tr>
<td>SUM</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
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\[ \hat{\Gamma} = \sum_{i=1}^{9} RRI_i \cdot E_i = (6.2371, 7.2371, 7.8866, 8.1855) \]
\[ ORI = \frac{\Gamma}{A} = \frac{10.7354}{1.4705} = 7.3004 \]

Which can be classified to be within a dangerous regime as shown in Figure 5 and Table III.

<table>
<thead>
<tr>
<th>Debris-Flow Hazards</th>
<th>Sen-Mu Village</th>
<th>Hua-San Village</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORI-AHP</td>
<td>8.0552</td>
<td>7.7881</td>
</tr>
<tr>
<td>ORI-FAHP (Using Triangular Fuzzy Numbers)</td>
<td>(3.32 %)*</td>
<td>(3.64 %)*</td>
</tr>
<tr>
<td>ORI-FAHP (Using Trapezoidal Fuzzy Numbers)</td>
<td>7.6443</td>
<td>7.3624</td>
</tr>
<tr>
<td>(5.10 %)*</td>
<td>(5.47 %)*</td>
<td></td>
</tr>
<tr>
<td>ORI-FAHP (Using Generalized Exponential Fuzzy Numbers)</td>
<td>7.5836</td>
<td>7.3004</td>
</tr>
<tr>
<td>(5.85 %)*</td>
<td>(6.26 %)*</td>
<td></td>
</tr>
</tbody>
</table>

\[ *\text{Deviation(\%)} = \frac{\text{AHP} - \text{FAHP}}{\text{AHP}} \times 100\% \]

In Figure 5 and Table III it can be obviously observed that only a small deviation (within 3% ~ 7%) exists between AHP and three FAHPs results. Furthermore, the results of risk indices predicted by AHP model are higher than those obtained from two FAHP models. This means the results of two FAHPs model are a little conservative. In addition, using nonlinear (generalized exponential) fuzzy numbers leads to wider vagueness of judgment and thus results in smaller overall risk indices than using linear (triangular and trapezoidal) fuzzy numbers. However, the prediction for these two real debris-flow hazards is nearly the same and all are within the “dangerous” range assessed using AHP and three FAHPs models.

V. EFFECT OF VAGUENESS OF GENERALIZED EXPONENTIAL FUNCTIONS

Then effect of different vagueness of generalized exponential functions on the predicted values will be studied.
Here two kinds of width of vagueness will be employed for comparison. The first generalized exponential function (mark by 1) is described as Eq. (3) and Figure 1 and 2. The second generalized exponential function is described by Eq. (3) and Figure 6 and 7. Following this concept, the example of definition of the second generalized exponential fuzzy number \( \tilde{a} = \tilde{\lambda} = (a, b, c, d) = (1, 2, 4, 5) \) and its inverse \( \tilde{a}^{-1} = \frac{1}{3} \cdot (\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a}) = (\frac{1}{5}, \frac{1}{4}, \frac{1}{2}, \frac{1}{1}) \).

The analysis procedure of risk assessment of debris-flow hazards is the same as Sec. IV with only the difference on variables with the values of fuzzy numbers involved, such as judgment matrices, Relative Risk Impact (RRI), judgment vector \( \{E\} \), and Overall Risk Index (ORI), etc.

Accordingly, the Relative Risk Impact (RRI) for the FAHP using the second generalized exponential fuzzy numbers becomes

\[
\{\text{RRI}\} = [M]_{C \rightarrow B} \cdot \begin{bmatrix} w^n_A \end{bmatrix}_{B \rightarrow A}
\]

The results are shown in Figure 8 in which RRI using FAHP with two different generalized exponential fuzzy numbers are depicted simultaneously for comparison. Only a little deviation exists between these two results.
Figure 8 Diagram for relative risk impact (RRI) for influence factors of debris-flow hazards using two FAHPs with different generalized exponential fuzzy numbers

The overall risk indices of the debris-flow occurred in Sen-Mu Village and Hua-San Village, respectively, predicted by FAHP using the second generalized exponential fuzzy numbers become

\[
\vec{\Gamma} = \sum_{i=1}^{9} RRI_i \cdot E_i = (6.0244, 7.0244, 8.3240, 8.6236)
\]

\[
ORI = \Gamma = \frac{Q_i}{A} = \frac{15.7494}{2.1211} = 7.4252
\]

\[
\vec{\Gamma} = \sum_{i=1}^{9} RRI_i \cdot E_i = (5.8744, 6.8744, 8.0369, 8.3365)
\]

\[
ORI = \Gamma = \frac{Q_i}{A} = \frac{14.2945}{1.9841} = 7.2047
\]

Which can be classified to be within a dangerous regime as shown in Figure 9. It shows that the wider the vagueness involved in the generalized exponential fuzzy numbers yield the lower the predicted values of risk assessment.

Figure 9 Diagram for overall index (ORI) of risk assessment of tested debris-flow hazards occurred at Sen-Mu Village and Hua-San Village using two FAHPs with different generalized exponential fuzzy numbers

VI. CONCLUDING REMARKS

A FAHP model using generalized exponential fuzzy numbers for risk assessment of debris-flow hazards has been successfully tested. The findings are:

1) the ranking of importance of influence factors to overall risk obtained from AHP and FAHP models are the same. Accumulated rainfall (AR), duration of rainfall (DR), and averaged slope angle (SA) plays the first three leading important influence factors of debris-flow hazards;

2) FAHP using generalized exponential fuzzy numbers yields smaller overall risk indices than FAHP using triangular and trapezoidal fuzzy numbers and the reason is due to the wider vagueness involved in the nonlinear(generalized exponential) fuzzy numbers than the linear (triangular and trapezoidal) fuzzy numbers;
3) Both three kinds of FAHP using linear (triangular and trapezoidal) and nonlinear (generalized exponential) fuzzy numbers predict smaller overall risk indices than the AHP model which using crisp judgment matrices.

4) The wider the vagueness involved in the generalized exponential fuzzy numbers yield the lower the predicted values of risk assessment.

REFERENCES