A Comprehensive Study on Mutual Relationship in Social Networks

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Abstract - In social networks the relationship between two nodes may be continuous of discontinuous over a period of time. Continuous means the relationship between two nodes are still alive whereas discontinuous means the relationship between two nodes does not exist. Graphically it means the relationship between two nodes are continuous when there is some connecting edge and relationship is called discontinuous when there is no edge available between two nodes. In the present paper the authors have made a thorough study on mutual relationship among various nodes in social networks.

Keyword-- Social, continuous, discontinuous, mutual, edge ;

I. INTRODUCTION

A relationship between two nodes in a social networks is subset of an ordered pair in which two nodes are related to each other. Relationship among nodes are very much dependent on the behavior of the nodes. Two nodes may be closely related with each others. In the social networks, it is also observed that relationship among some nodes can be extends with structure like mutual relationship. A relationship among nodes may change over a period of time. Relationship among mutual nodes depend on connective status/ structure/ position of nodes. If some nodes become unfriendly (removed) from a mutual relationship then these relation may not be mutually related. For example, let N₁, N₂, N₃ are three nodes of social networks which are mutually related with each others as follows:

Fig : 1

Now, {N₁, N₂, N₃} are not mutually related. So removal of edges from nodes (unfriendship) plays an important roles to check the existence of mutual relationship. A relationship among nodes may be either private or public. The practical approach of mutual relationship in social networking to send the message from one node to another node at any time through a path or via. some path. This path may be either public or private. So, in the above diagram when the relationship among two nodes N₂ and N₃ are disconnected (i.e. edge are removed ) then interconnecting node N₁ of N₂ and N₃ plays an important role and their behaviour of connection (public or private) with respective nodes will impact the relationship of node N₂ and N₃. In the present work the authors will study in detail the different cases of relationship after disconnection of relationship among some nodes.

II. MATHEMATICAL DESCRIPTION OF CONNECTIVITY OF NODES IN SOCIAL NETWORK

A. Social network [1,2]

A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, likings or disliking, or relationships of beliefs, knowledge or prestige. Social network analysis views social relationships in terms of network theory consisting of nodes and ties (also called edges, links, or connections). Nodes are the individual actors within the networks, and ties are the relationships between the actors. Hence, mathematically, social networking can be defined as the collection of socially connected elements/objects.
i.e., set $S=\{\text{social elements : social elements are connected}\}$

In a social network $S$, consider two nodes (i) $\text{node}_1$ represented by $N_1$ and (ii) $\text{node}_2$ represented by $N_2$ and consider a function $f$ defined between node $N_1$ and node $N_2$

$$f: N_1 \rightarrow N_2$$

Such that message send by node $N_1$ is received by node $N_2$ i.e., if $m_1,m_2,m_3,\ldots,m_n$ be the message send by the node $N_1$ to $N_2$, then $f(m_1), f(m_2), \ldots, f(m_n)$ will be message received by $N_2$

$$f(m_1), f(m_2), \ldots, f(m_n)$$

i.e., set of send message by node $N_1 \rightarrow$ set of received message by node $N_2$.

**B. Definition 1 [3]**

A social network is modeled as a graph $G(V, E)$, where $V$ represents a set of users embedded in a social context, and the edge set $E=\{(x, y) | x, y \in V\}$ represents friendship among users. An edge $e=(x, y)$ is added to $E$ when a friend request from $x$ to $y$ or from $y$ to $x$ is accepted. In SNSs (social networking sites), such as Facebook and LinkedIn, edges are usually undirected. For each user $u (u \in V)$, the set $F(u)=\{ x | x \in V, (x,u) \in E\}$ represents the friend list of $u$. Note, for each edge $e=(x, y)$, $x \in F(y)$ and $y \in F(x)$.

The mutual friends between two users can be defined as follows:

**C. Definition 2 [3][8]**

Given two users $x$ and $y (x, y \in V)$, we define the set of the mutual friends, $MF(x, y)$, between them as

$$MF(x, y) = F(x) \cap F(y)$$

Here, $MF(x, y)$ stands for mutual friendship between $x$ and $y$.

Intuitively, the definition of the mutual friend has two properties:

- Given $x, y, z \in V$, $y \in MF(x, z) \Leftrightarrow y \in F(x)$ or $y \in F(z)$

- Given $x, z$, $MF(x, z) = MF(z, x)$; i.e., mutual friendship is irreflexive and symmetric.

The basic needs of a node in social area networks are to maintain the relationship among all other nodes. The relationship between two nodes may be directly or indirectly connected.

### III. Mathematical Study of Some Relationship Among Nodes After Unfriendship in Mutual Relationship in Social Network

**A. Example of a mutual relationship**

Let node $N_1$ and $N_2$ are two node which are connected and $M_1, M_2, \ldots, M_n$ are their mutual nodes.

![Diagram](Fig:3)

Here $N_1$ and $N_2$ are two node which are connected with their mutual nodes.

Now if some nodes becomes unfriendly (disconnect) with other members of social networks. The following types of cases may occurs after disconnection of relationship of mutual nodes.

**case(i)**Some mutual nodes may be disconnected with the nodes $N_2$ i.e., nodes $N_2$ unfriends some mutual nodes

![Diagram](Fig:4)
Remarks: (a) Node $M_1$ is direct related with node $N_1$ and Node $M_1$ depends on node $N_1$ with respect to getting public message from remaining node $M_1, M_2, \ldots, M_n$ and node $N_2$. 

(b) Node $M_1, M_2, \ldots, M_n$ may establish relation with each other via Node $N_1$ and Node $N_2$. So, mutual nodes depend on the behaviour (public or private relationship) of Node $N_1$ and Node $N_2$. Mutual nodes may be publically or privately connected with node $N_1$ or node $N_2$. 

(c) Node $M_1$ may establish relationship (with respect to sending message) with node $N_1$ if node $N_1$ can share message (which is received by node $M_1$) to all remaining members.

case(ii) Some mutual nodes may be disconnected with the nodes $N_1$, i.e., Nodes $N_1$ unfriends some mutual nodes:

![Diagram 1](image1)

![Diagram 2](image2)

Remark: (a) Node $M_n$ is direct related with node $N_1$ and node $N_2$ depend on node $N_1$ with respect to getting public message from remaining node $M_1, M_2, \ldots, M_{n-1}$ and node $N_1$.

(b) Node $M_1, M_2, \ldots, M_n$ may establish relation with each other via Node $N_1$ and Node $N_2$. So, mutual nodes depend on the behaviour (public or private relationship) of Node $N_1$ and Node $N_2$ to establish the relationship.

Mutual nodes may be publically or privately connected with node $N_1$ or node $N_2$. 

(c) Node $M_n$ may establish relationship (with respect to sending or receiving message) with node $N_1$, $M_1, M_2, \ldots, M_{n-1}$ via the node $N_2$. Node $N_2$ may share message (which is received by node $M_n$) to all remaining members.

case(iii) Nodes $N_1$ and $N_2$ may be disconnected i.e., either nodes $N_1$ unfriends nodes $N_2$ or node $N_2$ unfriends node $N_1$.

![Diagram 3](image3)

Remark: (a) Node $N_1$ and Node $N_2$ are directly related with node $M_1, M_2, \ldots, M_n$. But Node $N_1$ and node $N_2$ depend on node $M_1, M_2, \ldots, M_{n-1}$, $M_n$ with respect to getting public message from node $M_1, M_2, \ldots, M_{n-1}, M_n$.

(b) Node $M_1, M_2, \ldots, M_n$ may establish relation with each other via Node $N_1$ and Node $N_2$. So, mutual nodes depend on the behaviour (for public or private relationship) of Node $N_1$ and Node $N_2$. Mutual nodes may be publically or privately connected with node $N_1$ or node $N_2$.

case(iv) Some mutual nodes may be disconnected with the nodes $N_1$ and $N_2$, i.e., nodes $N_1$ and node $N_2$ unfriend some mutual nodes.
Remarks: (a) Node $M_1$ is isolated.
(b) Node $M_2, \ldots, M_n$ may establish relation with each other via Node $N_1$ and Node $N_2$. So, mutual nodes depend on the behaviour (for public or private relationship) of Node $N_1$ and Node $N_2$. Mutual nodes may be publically or privately connected with node $N_1$ or node $N_2$.

case(v) Some mutual nodes are connected with node $N_1$ and some node are connected with node $N_2$, i.e., all mutual nodes are either connected with node $N_1$ or $N_2$. Node $N_1$ and $N_2$ are disconnected

Remarks: (a) Relationship between all nodes will be exist if relationship among all nodes are public
(b) Relationship between all nodes may not exist if relationship among some nodes are private.

B. Example of another mutual relationship where all mutual nodes are connected with each other

Remarks: (a) A single node may be able to send message to all nodes in public relationship
(b) Mutual nodes are independent with node $N_1$ and node $N_2$

case(i) Some mutual nodes are connected with each other. So remaining mutual nodes may be disconnected

Remarks: (i) A mutual node may not be able to send message to all nodes in directly. It will send the message via node $N_1$ and $N_2$. But mutual nodes may be publically or privately connected with node $N_1$ or node $N_2$. So relationship among all nodes dependent on their types of connectivity (public or private) with node $N_1$ or node $N_2$. 
IV. IMPORTANCE OF UNFRIENDSHIP IN SOCIAL NETWORKS

A. Understanding states of operation may be done to establish relationship (i.e., sending/receiving message) among all nodes after disconnection

The following states of operation are done to establish the relationship with nodes
(a) Initial states - The initial states state about which nodes want to send the message. Initial states is related with starting node.
(b) State space - It is description of all possible states (paths) reachable from the initial nodes. Please note that this state space forms a graph in which nodes are states and the edge represents actions.
(c) Goal test - The destination nodes are achieved by some path. This stage determines whether destination nodes is found from the initial (starting) nodes. The path may be public or private.
(d) Path Cost - It is a function that assigns a numeric cost to each path.

B. Understanding path or node dependency for establishing relationship after unfriendship

A relationship among nodes depends on the behaviour (public or private) of nodes. Two nodes may be publically or privately related with each other. A node can monitor all path to establish the relationship. After unfriendship some edges will be removed. Therefore a node has to search a path or a node through which it can be able to establish the relationship with its desirable nodes.

C. Understanding how much nodes are effected with disconnection

A mutual relationship among nodes of social networks may be affected by disconnection of relation among nodes. Some nodes or group of nodes will becomes isolated. There may exist some nodes which will not get path to establish the relationship.

D. Linear span of message

A single nodes can send message to all nodes if there is a path (public relationship) to connect all nodes.

E. Optimal path to connect all nodes

When the relationship between source node to destination nodes are established via some nodes or path then each path will assigns with some numeric cost.

So optimal path is that path in which optimal cost from source node to destination nodes are taken. During unfriendship some message may be send from one node to another node via some interconnecting nodes.

F. Conditional approach to establish relationship

Two nodes of social networks are related by sending and receiving message. When some edges are removed from some nodes they become disconncted. Therefore, message passing from one disconnected nodes to another disconnected nodes may depends on their interconnection nodes. So there is need to check the status of related nodes which will be as follows:
IF < relationship among two nodes are public > THEN < message can be passed to related to that two nodes >

G. Logical approach to establish relationship

Relationship among nodes of social networks depends on structures of connectivity of nodes. After unfriendship of relationship among two nodes, the structure of relationship will changed. Then the following logical approach may be done:

<table>
<thead>
<tr>
<th>Logic theory</th>
<th>Remarks/example</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOT</td>
<td>Source node want to send message to destination nodes but there are some interconnecting nodes are privately connected and they do not forward private message.</td>
</tr>
<tr>
<td>AND</td>
<td>A node can send message to all nodes (Which are publically connected) via some path.</td>
</tr>
<tr>
<td>OR</td>
<td>A node have more than one path to send message to another nodes of social networks</td>
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</tbody>
</table>

V. CONCLUSION

A relationship in social networks may vary with time. The number of friends of a nodes of social networks may either increase or decrease over a period of time.
When a mutual relationship among nodes established and if every nodes have good (nodes likes each other) relationship then it will be fine but if relationship among some nodes are not good (i.e one or more nodes disliking each other) over a period of time then relationship among these nodes may be hampered and they will make a disconnection. Now, the relationship of these disconnected nodes will depends on the nature of their interconnected nodes. If there is path(public relation) exist between these disconnected nodes message sending and receiving may be done. And if the path is in private relation, message sending and receiving may not be done between these nodes. So overall relationship depends on interconnected nodes which are working as bridge with disconnected nodes.

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