On (C,2)(E,Q) Product Means of Fourier Series and its Conjugate Series

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Abstract: In this paper introduces the concept of (C,2)(E,q) product operators and establishes two new theorems on (C,2)(E,q) product summability of Fourier Series and its conjugate Series. The results obtained in the paper further extend several known results on linear operators.

Keywords And Phrases: (C,2) summability, (E,q) summability, (C,1)(E,q) product summability, Fourier series, conjugate Fourier series, Lebesgue integral.

I. Introduction


Definition and notation

Let \( \sum_{n=0}^{\infty} u_n \) be a given infinite series with \( s_n \) for its \( n^{th} \) partial sum.

Let \( \{t_n^{(E,q)}\} \) denote the sequence of (E,q) means of the sequence \( \{s_n\} \). If the (E,q) transform of \( s_n \) is defined as

\[
t_n^{(E,q)}(f;x) = \frac{1}{(q+1)n} \sum_{k=0}^{n} \binom{n}{k} q^{n-k} S_k(f;x) \to s \text{ as } n \to \infty
\]  

the series \( \sum_{n=0}^{\infty} u_n \) is said to be summable to the number \( S \) by the (E,q) method (Hardy[10]).

Let \( \{t_n^{(C,2)}\} \) denote the sequence of (C,2) mean of the sequence \( \{s_n\} \). If the (C,2) transform of \( s_n \) is defined as

\[
t_n^{(C,2)}(f;x) = \frac{2}{(n+1)(n+2)} \sum_{k=0}^{n} (n-k+1) S_k(f;x) \to s \text{ as } n \to \infty
\]

the series \( \sum_{n=0}^{\infty} u_n \) is said to be summable to the number \( S \) by the (C,2) method (Cesàro method).

Thus if

\[
t_n^{(C,2)(E,q)}(f;x) = \frac{2}{(n+1)(n+2)} \sum_{k=0}^{n} (n-k+1) \frac{1}{(q+1)k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} S_v(f;x) \to s \text{ as } n \to \infty
\]
Where \( \{t_n^{(c,2)(E,q)}\} \) denote the sequence of \((c,2)(E,q)\) Product mean of the sequence \( s_n \), the series \( \sum_{n=0}^{\infty} u_n \) is said to be summable to the number \( S \) by the \((C,2)(E,q)\) method.

We observe that \((C,2)(E,q)\) method is regular.

Let \( f \) be a \( 2\pi \)-Periodic and Lebesgue integrable function, The Fourier Series associated with \( f \) at a point \( x \) is defined by

\[
f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \equiv \sum_{n=0}^{\infty} A_n(x) \quad (4)
\]

with partial sum \( S_n(f; x) \).

The conjugate Series of Fourier series (4) of \( f \) is given by

\[
\sum_{n=1}^{\infty} (a_n \cos nx - b_n \sin nx) \equiv \sum_{n=0}^{\infty} B_n(x) \quad (5)
\]

With Partial sums \( S_n(f; x) \).

Throughout this paper, we will call (5) as conjugate Fourier series of function \( f \).

We use the following notations;

\[
\phi(t) = \phi(x, t) = (x+t) + f(x+t) - 2f(x)
\]

\[
\Psi(t) = \Psi(x, t) = f(x+t) - f(x-t)
\]

\[
k_n(t) = \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} (\frac{k}{v}) q^{k-v} \frac{\sin((v+\frac{1}{2})t)}{\sin(\frac{1}{2})} \right]
\]

\[
k_{n/2}^{(1/2)}(t) = \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} (\frac{k}{v}) q^{k-v} \frac{\cos((v+\frac{1}{2})t)}{\sin(\frac{1}{2})} \right]
\]

II. MAIN THEOREMS

2.1 Theorem. Let \( \{C_n\} \) be a non-negative, monotonic, non-increasing sequence of real constants such that

\[
C_n = \sum_{\nu} C_{\nu} \rightarrow \infty \text{ as } n \rightarrow \infty
\]

If \( \phi(t) = \int_{0}^{t} \phi(u)du = o \left[ \frac{t}{a(t)C_1} \right] \text{ as } t \rightarrow +\infty \quad (6)
\]

Where \( a(t) \) is a positive, monotonic and non-increasing function \( t \) and

\[
\log(n) = O[\alpha(n)C_1], \text{ as } n \rightarrow \infty \quad (7)
\]

Then the Fourier series (4) is summable \((C,2)(E,q)\) to \( f(x) \).

2.2 Theorem. Let \( \{C_n\} \) be a non-negative, monotonic, non-increasing sequence of real constants such that

\[
C_n = \sum_{\nu} C_{\nu} \rightarrow \infty \text{ as } n \rightarrow \infty
\]
If \( \phi(t) = \int_0^t |\phi(u)|du = o\left[\frac{t}{\alpha(t)c_0}\right] \) as \( t \to +\infty \) \( (8) \)

Where \( \alpha(t) \) is a positive, monotonic and non-increasing function of \( t \),

\[
(1 + q)^t \sum_{k=1}^n \frac{(n-k+1)}{(1+q)^k} = \frac{\pi}{2} O(n + 1)(n + 2) \tag{9}
\]

And condition (7) holds then the conjugate Fourier series (5) is summable \((C,2)(E,q)\) to

\[
\tilde{f}(x) = -\frac{1}{2\pi} \int_0^{2\pi} \phi(t) \cot\left(\frac{t}{2}\right) dt
\]

At every point where this integral exists.

III. LEMMAS

Lemma 1. For \( 0 \leq t \leq \frac{1}{n} \), \( |K_n(t)| = O(n+1) \) or \( O(n) \); \( \sin nt \leq \text{nsint} \); \( |\cos nt| \leq 1 \)

Proof: For \( t \leq \frac{1}{n} \), \( \sin nt \leq \text{nsint} \)

\[
|K_n(t)| \leq \frac{1}{\pi(n+1)(n+2)} \left| \sum_{k=0}^{n} \sum_{v=0}^{k} \left( \frac{n-k+1}{(1+q)^k} \right) q^{k-v} \sin(v + \frac{1}{2}) \right|
\]

\[
\leq \frac{1}{\pi(n+1)(n+2)} \left| \sum_{k=0}^{n} \sum_{v=0}^{k} \left( \frac{n-k+1}{(1+q)^k} \right) q^{k-v} \frac{(2v+1)\sin t}{\sin \frac{t}{2}} \right|
\]

\[
\leq \frac{1}{\pi(n+1)(n+2)} \left| \sum_{k=0}^{n} \left( \frac{n-k+1}{(1+q)^k} \right) (2k+1) \sum_{v=0}^{k} \left( \frac{k}{v} \right) q^{k-v} \right|
\]

\[
\leq \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} [(n-k+1)(2k+1)]
\]

\[
\leq \frac{n+1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} (2k+1) - \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} [k(2k+1)]
\]

\[
\leq \frac{1}{\pi(n+2)} \sum_{k=0}^{n} (2k+1) - \frac{1}{\pi(n+1)(n+2)} \left[ 2 \sum_{k=0}^{n} k^2 + \sum_{k=0}^{n} k \right]
\]

\[
\leq \frac{(n+2)^2}{\pi(n+2)} - \frac{1}{\pi(n+1)(n+2)} \left[ \frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2} \right]
\]

\[
\leq \frac{(n+2)^2}{\pi(n+2)} - \frac{n(2n+1)}{3\pi(n+2)} - \frac{n}{2\pi(n+2)}
\]

\[
\leq \frac{2n^2 + 7n + 6}{6\pi(n+2)} = O(n)
\]
Lemma 2. For \( \frac{1}{n} \leq t \leq \pi \), \( |K_n(t)| = O \left( \frac{1}{t^2} \right) \), \( \sin \left( \frac{t}{2} \right) \geq \frac{t}{2} \) and \( \sin n t \leq 1 \)

Proof:- For \( \frac{1}{n} \leq t \leq \pi \), applying Jordan’s lemma, \( \sin \left( \frac{t}{2} \right) \geq \frac{t}{2} \) and \( \sin n t \leq 1 \)

\[
|K_n(t)| \leq \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{n-k+1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\sin(v + \frac{1}{2})}{\sin \frac{t}{2}} \right]
\]

\[
\leq \frac{(n+1)}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{1}{(t^2)} \right] - \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{k}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{1}{(t^2)} \right]
\]

\[
= \frac{2}{\pi(n+2)} \sum_{k=0}^{n} \left[ \frac{1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \right] - \frac{1}{\pi t(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{k}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \right]
\]

\[
= \frac{2}{\pi(n+2)} \sum_{k=0}^{n} \frac{1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} - \frac{1}{\pi t(n+1)(n+2)} \sum_{k=0}^{n} \frac{k}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v}
\]

\[
= \frac{2(n+1)}{\pi t(n+2)} - \frac{1}{2\pi t(n+1)(n+2)} \left[ \sum_{k=0}^{n} \frac{n(n+1)}{2} \right]
\]

\[
= \frac{2(n+1)}{\pi t(n+2)} - \frac{n}{2\pi t(n+2)}
\]

\[
= O \left( \frac{1}{t^2} \right)
\]

Lemma 3. For \( 0 \leq t \leq \frac{1}{n} \), \( \overline{K_n}(t) = O \left( \frac{1}{t^3} \right) \), \( \sin \left( \frac{t}{2} \right) \geq \frac{t}{2} \) and \( |\cos nt| \leq 1 \)

Proof:- For \( 0 \leq t \leq \frac{1}{n} \), \( \sin \left( \frac{t}{2} \right) \geq \frac{t}{2} \) and \( |\cos nt| \leq 1 \)

\[
|\overline{K_n}(t)| \leq \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{n-k+1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \cos \left( \frac{v + \frac{1}{2}}{t} \right) \right]
\]

\[
\leq \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{n-k+1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \cos \frac{v + \frac{1}{2}}{t} \right]
\]

\[
\leq \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{n-k+1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{1}{(t^2)} \right]
\]

\[
\leq \frac{2}{\pi t(n+1)(n+2)} \sum_{k=0}^{n} \left[ \frac{n-k+1}{(1+q)^k} \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \right]
\]
Lemma 4. For $0 \leq \alpha \leq b \leq \infty$, $0 \leq t \leq \pi$ and any $n$, 

$$|k_n(t)| = O \left( \left( k \left( \frac{t}{t} \right) + k \left\{ \frac{2}{\pi(t(n+1)(n+2))} \sum_{k=\tau}^{n} \frac{(n-k+1)}{(1+q)^k} \right\} \right) \right)$$

Proof: For $0 \leq \frac{1}{n} \leq t \leq \pi$, $\sin \left( \frac{t}{2} \right) \geq \frac{t}{2}$

$$|k_n(t)| \leq \frac{1}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} \cos \left( \frac{\nu}{2} \right) \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} e^{i(\nu+\frac{1}{2})t} \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} e^{i\nu t} \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{\tau-1} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} e^{i\nu t} \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=\tau}^{n} \left( \frac{1}{n} \sum_{v=0}^{k} \left( k \right) q^{k-v} e^{i\nu t} \right) \right|$$

$$= k_1 + k_2$$

(10)

Where $\tau$ denoted the integral part of $\frac{1}{t}$.

Now Considering first term of (10)

$$|k_1| \leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{\tau-1} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} e^{i\nu t} \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{\tau-1} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} \right] e^{i\nu t} \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \left| \sum_{k=0}^{\tau-1} \left[ \frac{(n-k+1)}{(1+q)^k} \sum_{v=0}^{k} \left( k \right) q^{k-v} \right] \right|$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \sum_{k=\tau}^{n-1} \left( n - k + 1 \right)$$

$$\leq \frac{2}{\pi t(n+1)(n+2)} \sum_{k=0}^{\tau-1} (n+1) - \frac{2}{\pi t(n+1)(n+2)} \sum_{k=\tau}^{n-1} k$$
And

\[ |k_2| \leq \frac{2}{\pi(n+1)(n+2)} \sum_{k=1}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \max_{0 \leq m \leq k} \left| \sum_{\nu=0}^{k} q^{-\nu} e^{i\nu t} \right| \right] \]

Equation (12)

Combining (10) to (12), we get

\[ K_n(t) \leq k \left( \frac{1}{t} \right) + k \left\{ \frac{2}{\pi(n+1)(n+2)} (1 + q)^t \sum_{k=1}^{n} \frac{(n-k+1)}{(1+q)^k} \right\} \]

Equation (13)

IV. PROOF OF MAIN THEOREM

Following Titchmarsh [25] and using Riemann-Lebesgue theorem, \( S_n(f;x) \) of the series (1.4) is given by

\[ S_n(f;x) = \frac{1}{2\pi} \int_0^{\pi} \phi(t) \frac{\sin \left( n + \frac{1}{2} \right) t}{\sin \frac{t}{2}} \, dt \]

Using (1), the \((E,q)\) transform of \( S_n(f;x) \) is given by

\[ t_n^{(E,q)} - f(x) = \frac{1}{2\pi(1+q)^k} \int_0^{\pi} \phi(t) \left( \sum_{\nu=0}^{k} q^{-\nu} \frac{\sin \left( \nu + \frac{1}{2} \right) t}{\sin \frac{t}{2}} \right) \, dt \]

Now denoting \((C,2)(E,q)\) transform of \( S_n(f;x) \) by

\[ t_n^{(C,2)(E,q)} - f(x) = \frac{1}{\pi(n+1)(n+2)} \sum_{k=1}^{n} \left[ \frac{(n-k+1)}{(1+q)^k} \int_0^{\pi} \phi(t) \left( \sum_{\nu=0}^{k} q^{-\nu} \sin \left( \nu + \frac{1}{2} \right) t \right) \, dt \right] \]

In order to prove that the theorem, we have to show under our assumptions that

\[ \int_0^{\pi} \phi(t) K_n(t) \, dt = o(1) \text{ as } n \to \infty \]

For \( 0 < \delta < \pi \), we have

\[ \int_0^{\pi} \phi(t) K_n(t) \, dt = \left( \int_0^{\frac{\delta}{n}} \phi(t) + \int_{\frac{\delta}{n}}^{\delta} \phi(t) + \int_{\delta}^{\pi} \phi(t) \right) k_n(t) \, dt \]

\[ = I_1 + I_2 + I_3 \text{ (Say)} \]

Equation (14)
We consider,

\[ |l_1| \leq \int_0^{1/n} |\varphi(t)||k_n(t)|dt \]

= \( O(n) \left[ \int_0^{1/n} |\varphi(t)|dt \right] \) using lemma 1

= \( O(n) \left[ o \left( \frac{1}{n\alpha(n)C_n} \right) \right] \) by (6)

= \( o \left( \frac{1}{\log n} \right) \) by (7)

= \( o(1) \), as \( n \to \infty \) \hspace{1cm} (15)

Now we consider

\[ |l_2| \leq \int_{1/n}^{\delta} |\varphi(t)||k_n(t)|dt \]

= \( O \left[ \int_{1/n}^{\delta} |\varphi(t)| \left( \frac{1}{t} \right) dt \right] \) using lemma 2

= \( O \left[ \frac{1}{t} \varphi(t) \right]_{1/n}^{\delta} \]

= \( O \left[ \frac{1}{\alpha(n)C_n} \right]_{1/n}^{\delta} \)

Putting \( \frac{1}{t} = u \) in second term,

= \( O \left[ \frac{1}{\alpha(n)C_n} \right]_{1/n}^{\delta} + \int_{1/n}^{n} o \left( \frac{1}{u\alpha(u)C_u} \right) du \) by (6)

Using second mean value theorem for the integral in the second term as \( \alpha(n) \) is monotonic

= \( o(1) + o(1) \) as \( n \to \infty \)

= \( o(1) \), as \( n \to \infty \) \hspace{1cm} (16)

By Riemann- Lebesgue lemma and by regularity condition of the method of Summability,

\[ |l_2| \leq \int_{\delta}^{\pi} |\varphi(t)||k_n(t)|dt \]

= \( o(1) \), as \( n \to \infty \) \hspace{1cm} (17)

Combining (14) to (17) we have

\[ t^{(C2)(Eg)} - f(x) = o(1) \), as \( n \to \infty \]

This completes the proof of theorem 1.
Proof of Theorem. Let $\tilde{s}_n(f; x)$ denotes the partial sum of series (5).

Then following Lal[4] and using Riemann-Lebesgue Theorem, $\tilde{s}_n(f; x)$ of series (5) is given by

$$\tilde{s}_n(f; x) - \tilde{f}(x) = \frac{1}{2\pi} \int_0^\pi \psi(t) \frac{\cos\left(\frac{n+1}{2}t\right)}{\sin\frac{t}{2}} dt$$

Therefore using (5), the (E,q) transform of series (5) is given by

$$\tilde{\epsilon}^{(E,q)}_n - \tilde{f}(x) = \frac{1}{2\pi(1+q)^k} \int_0^\pi \psi(t) \left( \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\cos\left(\frac{v+1}{2}t\right)}{\sin\frac{t}{2}} \right) dt$$

Now denoting (C,2)(E,q) transform of $\tilde{s}_n$ is given by

$$\tilde{\epsilon}^{(C,2)(E,q)}_n - \tilde{f}(x) = \frac{1}{\pi(n+1)(n+2)} \sum_{k=0}^{n} \left\{ \int_0^\pi \psi(t) \left( \sum_{v=0}^{k} \binom{k}{v} q^{k-v} \frac{\cos\left(\frac{v+1}{2}t\right)}{\sin\frac{t}{2}} \right) dt \right\}$$

In order to prove the Theorem, we have to show that, under the hypothesis of theorem

$$\int_0^\pi \psi(t) \tilde{k}_n(t) dt = o(1) \quad \text{as} \quad n \to \infty$$

For $0 < \delta < \pi$, we have

$$\int_0^\pi \psi(t) \tilde{k}_n(t) dt = \int_0^{\frac{\pi}{2}} \psi(t) dt + \int_0^{\frac{\delta}{2}} \psi(t) dt + \int_\frac{\delta}{2}^\pi \psi(t) dt \tilde{k}_n(t) dt$$

$$= J_1 + J_2 + J_3 \quad (\text{Say}) \quad (18)$$

We consider,

$$|J_1| \leq \int_0^{\frac{\pi}{2}} |\psi(t)| \left| \tilde{k}_n(t) \right| dt$$

$$= O \int_0^{\frac{\pi}{2}} \left| \psi(t) \right| dt \quad \text{using lemma 3}$$

$$= O(n) \left[ \int_0^{\frac{\pi}{2}} \left| \psi(t) \right| dt \right]$$

$$= O(n) \left[ o\left(\frac{1}{\alpha(n)C_n}\right) \right] \quad \text{By (8)}$$

$$= o\left(\frac{1}{\alpha(n)C_n}\right) \quad \text{by (7)}$$

$$= o(1) \quad \text{as} \quad n \to \infty \quad (19)$$

Now,

$$|J_2| \leq \int_0^{\frac{\pi}{2}} |\psi(t)| \left| \tilde{k}_n(t) \right| dt$$

$$\leq \left[ k \int_0^{\frac{\delta}{2}} \left( n^k + 2 \right) \sum_{k=0}^{n-1} \frac{n^{k+1}}{-(1+q)^k} \right] \left| \psi(t) \right| dt \quad [\text{using lemma 4}]$$
\[
= 0 \left[ \int_{1/n}^{\delta} |\psi(t)| dt \right] \text{ by (9)}
\]
\[
= O \left[ \int_{1/n}^{\delta} |\psi(t)| dt + \int_{1/n}^{\delta} \frac{1}{t^2} \psi(t) dt \right]
\]
\[
= O \left[ \int_{1/n}^{\delta} \frac{1}{a(t)c(t)} dt + \int_{1/n}^{\delta} \frac{1}{1/a(u)c(u)} du \right]
\]
\[
= o \left[ \int_{1/n}^{\delta} \frac{1}{a(n)c(n)} + \int_{1/n}^{\delta} \frac{1}{\alpha(n)c(n)} \right] \text{ by (7)}
\]
\[
= o(1) + o(1) , \text{ as } n \to \infty
\]
\[
= o(1) , \text{ as } n \to \infty
\]
\[
\text{By Riemann – Lebesgue lemma and by regularity condition of } (C,2)(E,q) \text{ method of Summability,}
\]
\[
= o(1) , \text{ as } n \to \infty
\]
Combining (18) to (21) we have,
\[
\tilde{f}(x) = o(1) , \text{ as } n \to \infty
\]

This completes the proof of theorem 2.

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REFERENCES


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