Effect of Magnetic Parameter on Newtonian Fluids when Speed of the Plate is Less than the Fluid using Collocation Method

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Abstract— In this paper, we have studied the equation of Magnetohydrodynamic boundary layer model for the Newtonian case of power law fluid in the presence of transverse magnetic field when the speed of plate is less than the fluid. Here we have checked the effect of a magnetic parameter with increase of velocity parameter. The governing nonlinear differential equations are solved using Spline functions due to Blue. The prettiness of this method is, we can solve a nonlinear problem directly, without converting into linear form. Method description and graphical results are described.

Keywords– Power-law fluids, Magnetic field, Velocity parameter, nonlinear differential equation, Collocation method.

I. INTRODUCTION


II. PROBLEM FORMULATION

Here we studied the two-dimensional laminar boundary layer flow of a viscous, incompressible and electro conducting power law fluid past a continuously moving surface passing through with constant \( U_w \) in the same direction to the free stream velocity \( U_\infty \). The \( x \)-axis extends parallel to the plate and \( y \)-axis towards perpendicular to the \( x \)-axis. A magnetic field of uniform strength \( B_0 \) is applied in the positive \( y \)-direction, which produce the magnetic field in the \( x \)-direction. The boundary layer equations governing the flow in a power-law fluid are

\[
\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} - \frac{\sigma B_0^2}{\rho} u \tag{2}
\]

Where \( u, v \) are the velocity components along \( x \) and \( y \) coordinates, \( \tau_{xy} \) is the shear stress and \( \rho \) is the fluid density.

With the boundary conditions:

\[
y = 0: \quad u = U_w, \quad v = 0 \tag{3}
\]

\[
y = \infty: \quad u = U_\infty
\]

We apply power-law relation between the shear stress and the shear rate by

\[
\tau_{xy} = K \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} v = -\frac{\partial v}{\partial x}
\]
Where $\gamma \frac{\partial u}{\partial y}$ denotes the kinematic viscosity, $K$ is the consistency coefficient $\gamma = \frac{K}{\rho}$ and $n$ is the power-law index, for $n < 1$ the fluid is pseudo plastic, for $n = 1$ the fluid is Newtonian and for $n > 1$, dilatants fluid. The equation (2) becomes

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\partial}{\partial y} \left( \gamma \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0^2}{\rho} u \quad \text{(4)}$$

Introducing the stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x},$$

which satisfy the continuity equation (1). Govind R. Rajput et al [5] converted partial differential equation into nonlinear ordinary differential equations using Group theoretic method. They considered the following transformation:

$$\psi(x, y) = ax^a f(\eta), \quad \eta = b \frac{y}{x^b} \quad \text{(5)}$$

Where $a, b, \alpha$ and $\beta$ are real numbers, $\eta$ is similarity variable, $f(n)$ is the transformed dimensionless stream function.

Applying this similarity variable $\eta$ they derive

$$\psi_x = ax^{a-1}[\alpha f - \eta \beta f']$$

$$\psi_y = abf' x^{-\beta}$$

$$\psi_{yy} = ab^2 x^{-2\beta} f'$$

$$\psi_{yx} = abx^{a-\beta} \left[ \alpha f' - \beta f' - \beta \eta f' \right] \quad \text{(6)}$$

Using equation (5) along with (6) into (4) they get transformed into nonlinear ordinary differential of the form

$$\left( f^{(n+1)} - f' \right) - Mf' + \frac{1}{2} ff'' = 0 \quad \text{(7)}$$

with the transformed boundary conditions:

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad f'(\infty) = 1 \quad \text{(8)}$$

We consider the case $n = 1$ then the equation (7) becomes

$$f^{(3)} - Mf' + \frac{1}{2} ff'' = 0 \quad \text{(9)}$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad f'(\infty) = 1 \quad \text{(10)}$$

Where $\varepsilon = \frac{U_B}{U_{\infty}}$ is the velocity parameter and $M = \frac{\sigma B_0^2}{\rho U_{\infty}}$ is the magnetic parameter. Here note that when $0 < \varepsilon < 1$, the speed of the plate is less than the fluid. We are going to solve above equation (9) with boundary condition (10) using Spline Collocation method numerically.

### III. QUARTIC SPLINE BLUE METHOD

For three points boundary value problems, let $s_i(x)$ be quartic spline function in $[x_{i-1}, x_i]$. Conditions for natural splines are

(i) $s_i(x)$ Quartic in each subinterval $[x_{i-1}, x_i]$.  
(ii) $s_i(x_i) = y_i$, for $i = 0, 1, 2, \ldots, n$.  
(iii) $s_i(x_{i-1}), s_i(x_i), s'_i(x_i), s''_i(x_i)$ are continuous in $[x_{i-1}, x_i]$.  
(iv) $s'_i(x_0) = s''_i(x_n) = 0$.

Here third derivative of spline must be linear in $[x_{i-1}, x_i]$. So,

$$s''_i(x_i) = \frac{1}{h_i} \left[ (x_i - x)y_{i-1}^{(i)} + (x - x_{i-1}) y_{i}^{(i)} \right] \quad \text{(11)}$$

Where $h_i = x_i - x_{i-1}$ and $s''_i(x_i) = y''_i$.

Integrate (11) twice with respect to $x$
\[ s'_i(x) = \frac{1}{h_i} \left[ \frac{(x_i-x)^3}{6} y_{i-1} + \frac{(x-x_{i-1})^3}{6} y_i \right] \] (12)

\[ + c_i(x_i-x) + d_i(x-x_{i-1}). \]

Where \( s'_i(x_{i-1}) = y_{i-1} \) and \( s'_i(x_i) = y_i \) in (12), we get constants \( c_i \) and \( d_i \)

\[ c_i = \frac{1}{h_i} \left( y_{i-1} - \frac{h_i^2}{6} y_{i-1} \right) \] and \( d_i = \frac{1}{h_i} \left( y_i - \frac{h_i^2}{6} y_i \right) \)

So

\[ s'_i(x) = \frac{1}{h_i} \left[ \frac{(x_i-x)^3}{6} y_{i-1} + \frac{(x-x_{i-1})^3}{6} y_i \right] \]

\[ + \frac{1}{h_i} \left( y_{i-1} - \frac{h_i^2}{6} y_{i-1} \right) (x_i-x) + \frac{1}{h_i} \left( y_i - \frac{h_i^2}{6} y_i \right) (x-x_{i-1}). \] (13)

Integrate (13), once with respect to \( x \),

\[ s_i(x) = \frac{1}{h_i} \left[ -\frac{(x_i-x)^4}{24} y_{i-1} + \frac{(x-x_{i-1})^4}{24} y_i \right] \]

\[ - \frac{1}{h_i} \left( y_{i-1} - \frac{h_i^2}{6} y_{i-1} \right) \frac{(x_i-x)^2}{2} + \] (14)

\[ + \frac{1}{h_i} \left( y_i - \frac{h_i^2}{6} y_i \right) \frac{(x-x_{i-1})^2}{2} + e_i. \]

Take \( s_i(x_{i-1}) = y_{i-1} \), we get constants \( e_i \)

Where \( e_i = y_{i-1} - \frac{h_i^3}{8} y_{i-1} + \frac{h_i}{2} y_{i-1}. \)

Substitute \( e_i \) in (13), we get

\[ s_i(x) = \frac{1}{h_i} \left[ -\frac{(x_i-x)^4}{24} y_{i-1} + \frac{(x-x_{i-1})^4}{24} y_i \right] \]

\[ - \frac{1}{h_i} \left( y_{i-1} - \frac{h_i^2}{6} y_{i-1} \right) \frac{(x_i-x)^2}{2} + \]

\[ + \frac{1}{h_i} \left( y_i - \frac{h_i^2}{6} y_i \right) \frac{(x-x_{i-1})^2}{2} + e_i. \]

Here

\[ s'_i(x_i^-) = s'_{i+1}(x_{i+}^-) \] and for equal intervals we have,

\[ y_{i+1}^- - 2 y_i^- + y_{i-1}^- = \frac{h_i^2}{6} (y_{i+1}^- + 4 y_i^- + y_{i-1}^-) \] (15)

And for

\[ s_i(x_i^+) = s_{i+1}(x_{i+}) \] and for equal intervals we have,

\[ y_{i+1}^- - y_i^- = -\frac{h}{2} (y_i^- + y_{i-1}^-) + \frac{h^3}{24} (3 y_{i-1}^- - y_i^-) \] (16)

IV. SOLUTION USING SPLINE FUNCTION DUE TO BLUE

We Solve this equation (9) with boundary condition (10) for the case when \( 0 < e < 1 \). Here we have checked effect of magnetic parameter and velocity parameter.

Case (i) \( e = 0.2 \)

To obtain the spline solution, begin with an assumed function \( f(\eta) = 0.4 \eta^2 + 0.2 \eta \)

Which satisfy given boundary conditions (10). We find the solution of equation (9) along with boundary conditions (10) using the equations (15) and (16). Put \( h = 0.2 \) in (15), we get different values of \( y_i \) for \( i = 1, 2, 3, 4 \). To find the final solution we use (16) for different values of \( i = 1, 2, 3, 4 \) respectively.
Thus, we obtain the system of equations as follows

\[
\begin{align*}
    y_0 - y_1 &= -\frac{h}{2} [y_1 + y_0] + \frac{h^2}{24} [y_1^* - 3y_0^*] \\
y_1 - y_2 &= -\frac{h}{2} [y_2 + y_1] + \frac{h^2}{24} [y_2^* - 3y_1^*] \\
y_2 - y_3 &= -\frac{h}{2} [y_3 + y_2] + \frac{h^2}{24} [y_3^* - 3y_2^*] \\
y_3 - y_4 &= -\frac{h}{2} [y_4 + y_3] + \frac{h^2}{24} [y_4^* - 3y_3^*] \\
y_4 - y_5 &= -\frac{h}{2} [y_5 + y_4] + \frac{h^2}{24} [y_5^* - 3y_4^*]
\end{align*}
\]

Substitute \(y_i\) and \(y_i^*\) in above equations. We get five unknown and five equations. Solving that system using Matlab, we get solution graph as follows:

**Case (ii) \( \varepsilon = 0.6 \)**

Boundary conditions have been changed due to change in velocity parameter. So the assumed function also changed and the function is \( f(\eta) = 0.2\eta^2 + 0.6\eta \).

Thus using this assumed function and the equations (15) and (16), we get the solution of equation (9).

To find the solutions, we have to follow the process as explained in case (i). Solution graphs are as follows:

**Case (ii) \( \varepsilon = 0.8 \)**

The assumed function is \( f(\eta) = 0.1\eta^2 + 0.8\eta \).

Using this assumed function and equations (15-16), we get the solution of equation (9).

For this case also, follows the process like case (i). Graphical presentation of solution as follows:
V. RESULT AND DISCUSSION

The Graphical results show the significant effect of magnetic parameter on the displacement profile of the flow (Fig. 1, 3, 5). We can see that, displacement of fluid at particular point decrease as increase in magnetic parameter (Fig.2, 4, 6). This demonstrates that the increment in magnetic parameter shows remarkable change in velocity profile. Velocity profile decay as increase in magnetic parameter. Thus we can say that, velocity of MHD flow of Newtonian-power law fluid is transversely proportional to magnetic parameter. As increase of velocity parameter, displacement and velocity profile increase at particular point (Fig. 7, 8). That means that, velocity parameter is directly proportional to velocity of fluid and velocity of plate increase as increase in velocity parameter.

VI. CONCLUSION

In Newtonian power-law MHD flow, there is a remarkable effect of magnetic parameter and velocity parameter on flow. The Magnetic parameter is transversely proportional to the velocity of fluid and velocity profile directly promotional to a velocity of the fluid. Both parameters work oppositely on fluid flow. Speed of plate increase as an increase in velocity parameter.

We find the generalization of the blue method for third order problem. In this method no need to convert a nonlinear problem into a linear form, we can get the solution directly for the nonlinear form. Thus, without doing the experimental work, we can get the results for such types of problems and save the time and experimental cost.
REFERENCES


